LANL/IGPP Climate Study Group
04/10/2007

Three-Dimensional Radiative Transfer:

What is it?

&

Why does it matter?

Anthony B. Davis

Los Alamos National Laboratory

Space & Remote Sensing Group (ISR-2)

LA-UR 07-2467

NNSA

Los Alamos

… with ideas, papers, discussions, critiques from/by many colleagues. For starts …

Howard Barker (MSC)
Klaus Pfeilsticker (Heidelberg)
Qi-Long Min (SUNY-Albany)
Lee Harrison (SUNY-Albany)
Greame Stephens (CSU)
Dennis O'Brien (CSU)
Alex Kostinski (MTU)
Mike Larson (ARL)
Dave Winker (NASA-LaRC)
Yong Hu (NASA-LaRC)
Yuri Knyashikhin (BU)
Mike Hall (LANL/CCS-2)
Jim Morel (TAM, *ex-LANL*)
Igor Polonsky (CSU, *ex-LANL*)
Warren Wiscombe (BNL, GSFC)
Sasha Marshak (NASA-GSFC)
Bob Cahalan (NASA-GSFC)
Tamas Várnai (NASA-GSFC)
Lazaros Oreopoulos (GSFC)
Frank Evans (U of Co)
Bernard Mayer (DRL)
Eugene Clothiaux (PSU)

Over the past 9+ years, funding from:

* DOE's Multispectral Thermal Imager (MTI)
* NNSA/NA-22 programs
* ISR Division's PD- and TD-programs

* LDRD/ER, -/DR, and -/PRD
* NASA-GSFC Collaborations ("work of others")
* [SC/BER] ARM Program - Science Team PI

ARM
Atmospheric Radiation Measurement

Prologue

… followed by 4 acts

&

an epilogue

State-of-the-Art Conceptual Model for Radiative Transfer in Clouds

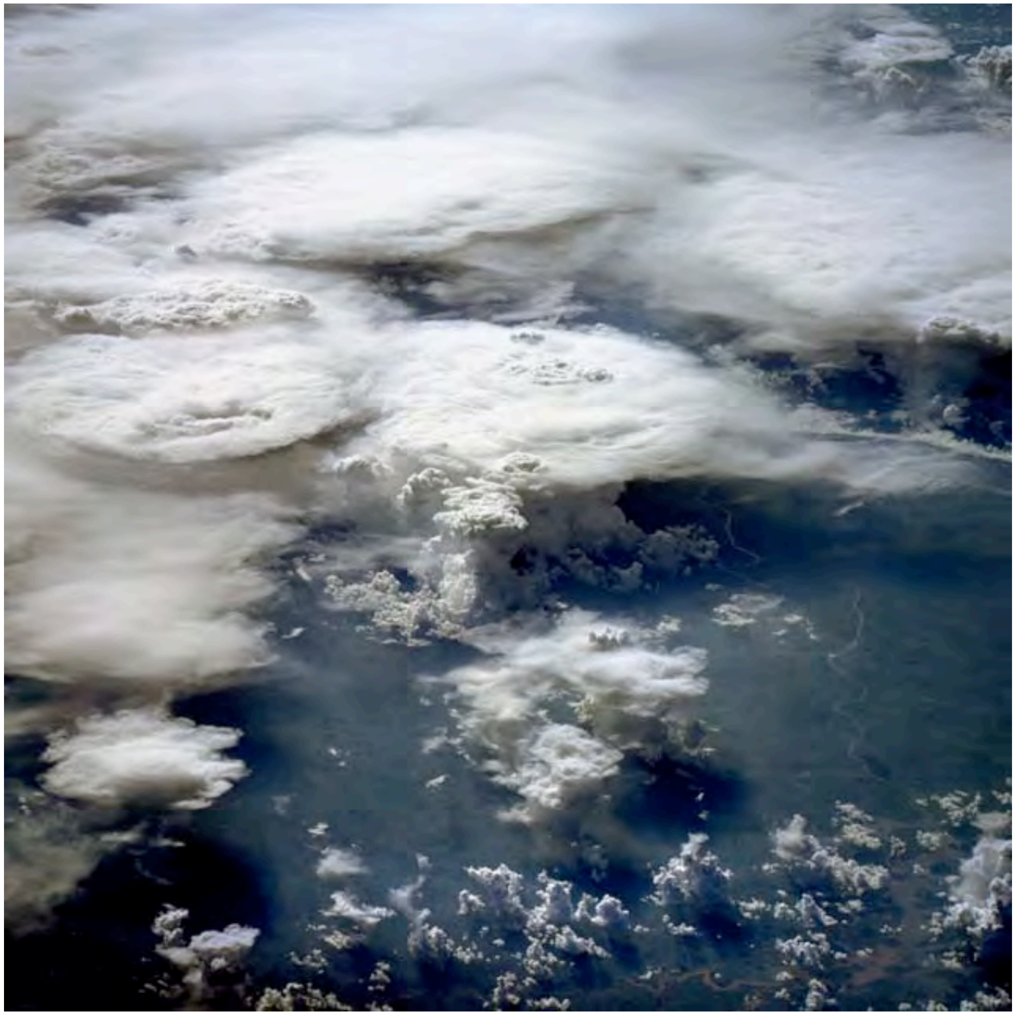
- To compute radiative fluxes inside most Global Climate Models (GCMs) radiation modules, as well as cloud system resolving models (CSRMs) and even cloud process models.
- To compute radiances, hence physical cloud properties, in *operational* cloud remote sensing schemes at NASA and Co, irrespective of pixel size.

This is a cloud!

NNSA

Los Alamos

Reality:



What we* compute versus what we* measure

$I_\lambda(\boldsymbol{x},\vec{\Omega})$, from 3D Radiative Transfer Equation (RTE), plus BCs.

$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} f(\lambda) \int_{\boldsymbol{x}\in \mathrm{S}\subseteq \Re^3} \chi(\boldsymbol{x}) \int_{4\pi} r(\vec{\Omega})\, I_\lambda(\boldsymbol{x},\vec{\Omega})\, \mathrm{d}\lambda \mathrm{d}\boldsymbol{x} \mathrm{d}\vec{\Omega}$$

E.g., BB actinic flux in a CSRM or LES cell :

$\lambda_{\min} = 0, \lambda_{\max} = \infty, f(\lambda) \equiv 1$

$$\chi_{\boldsymbol{x}^*}(\boldsymbol{x}) = \begin{cases} 1/\Delta x \Delta y \Delta z, & x^* \le x < x^* + \Delta x,\ y^* \le y < y^* + \Delta y,\ z^* \le z < z^* + \Delta z \\ 0, & \text{otherwise} \end{cases}$$

$r(\vec{\Omega}) \equiv 1$

*** "we" can be Nature**

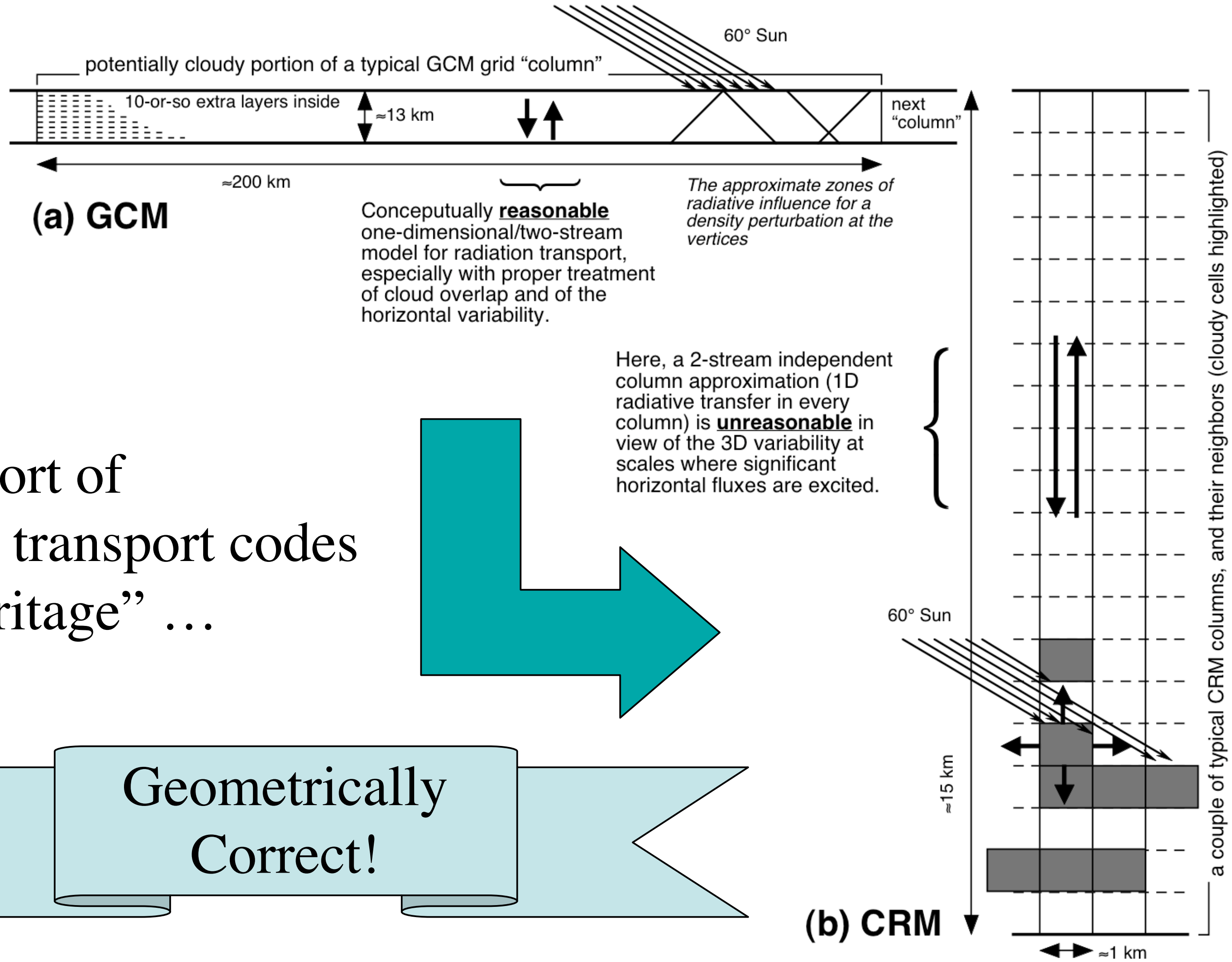
3D RT Problem Space

$I_\lambda(\boldsymbol{x},\vec{\Omega})$ $f(\lambda)\chi(\boldsymbol{x})r(\vec{\Omega})$	**Diagnostics** using radiances [samples of Ω]	**Energetics** using fluxes [sums over Ω]
"pixel" scales: <u>resolved</u> structure [samples of $\boldsymbol{x}$]	"Pixel/column adjacency" problems I3RC	3D radiative heating/cooling rates in CSRMs I3RC-Approx
"domain" scales: <u>unresolved</u> structure [sums over $\boldsymbol{x}$]	Large-footprint (or beam-filling) problems RAMI	GCM radiation parameterization schemes ICRCCM

ACT 1

"Good enough" 3D RT

In 3D RT, consider the inner and outer aspect ratios.

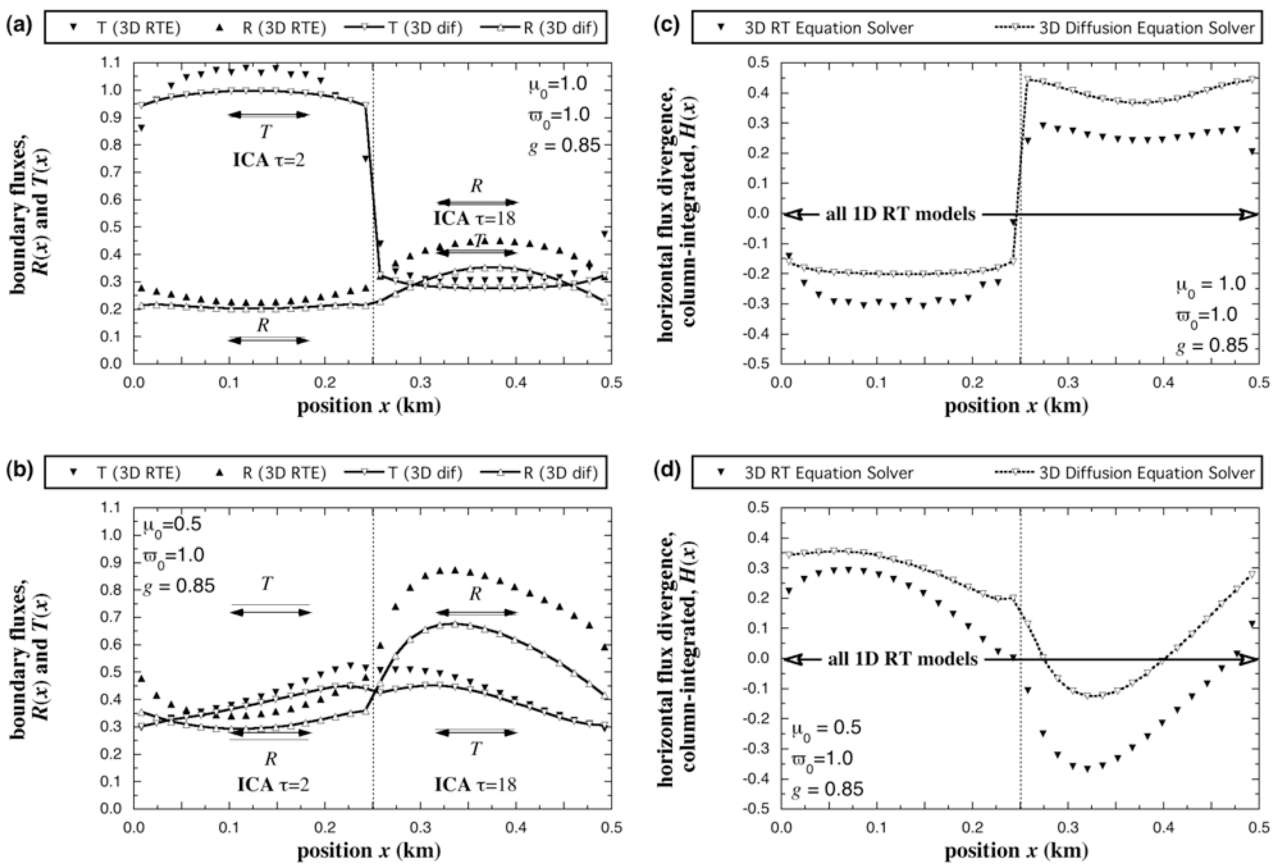


Transport of photon transport codes by “heritage” …

Geometrically Correct!

1D Independent Column Approximation (ICA) versus 3D Transport and Diffusion

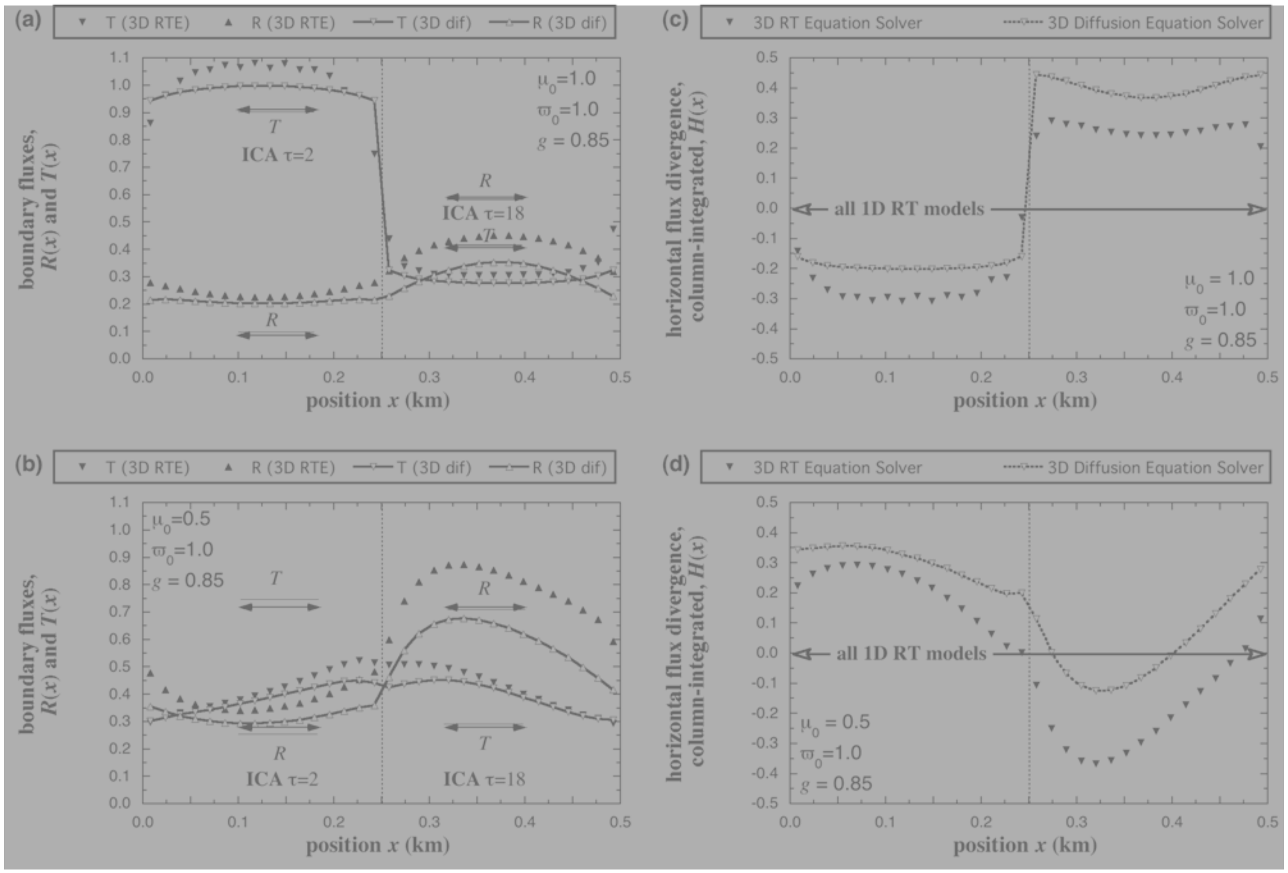
I3RC "Case 1" square-wave cloud: 0.25 km thick, alternating $\tau = 2{,}18$ every 0.25 km.

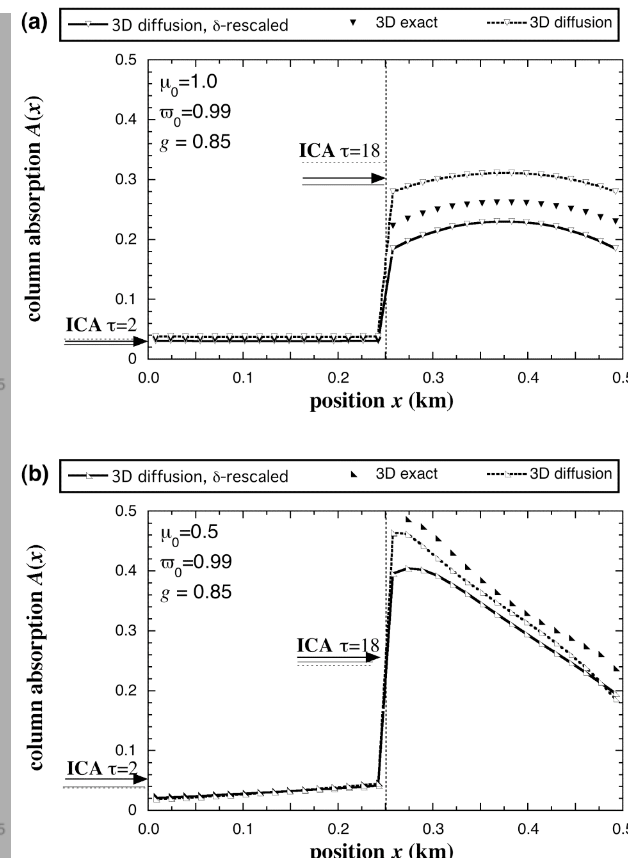


$\varpi_0 = 1$

1D Independent Column Approximation (ICA) versus 3D Transport and Diffusion

I3RC “Case 1” square-wave cloud: 0.25 km thick, alternating τ = 2,18 every 0.25 km.



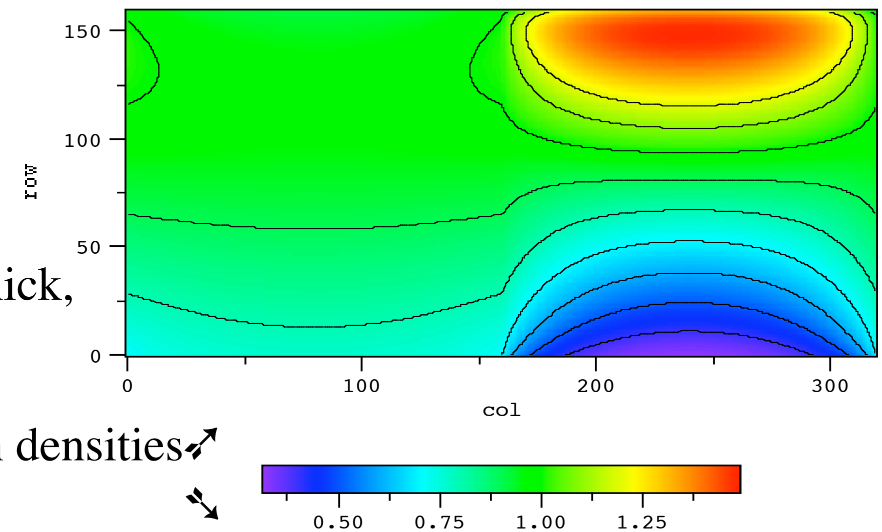


$\varpi_0 = 1$ | $\varpi_0 = 0.99$

3D RT in *O*(*N*) FLOPs: Computational Diffusion Theory

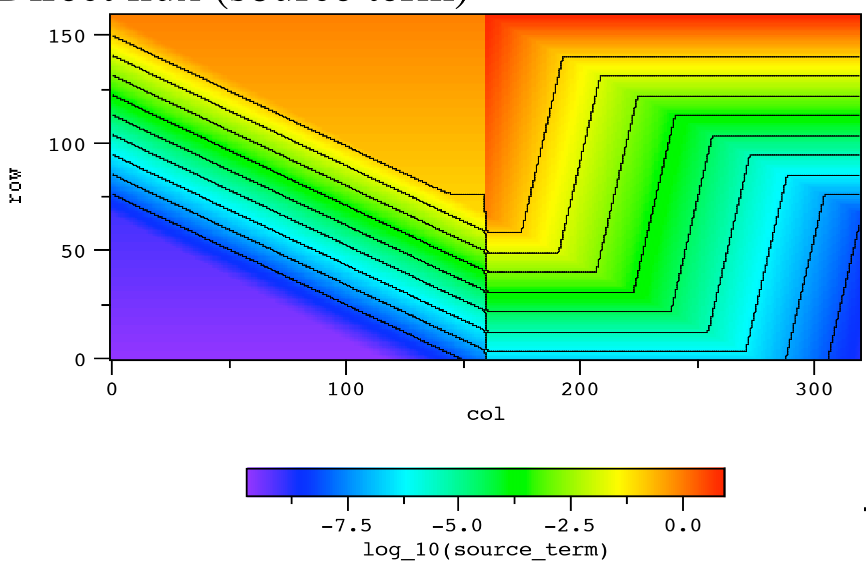
- **Joint work with <u>Mike Hall (CCS-2)</u>**
- **Ideal for stratus layers**
- **Multi-grid solver, parallelizing code**

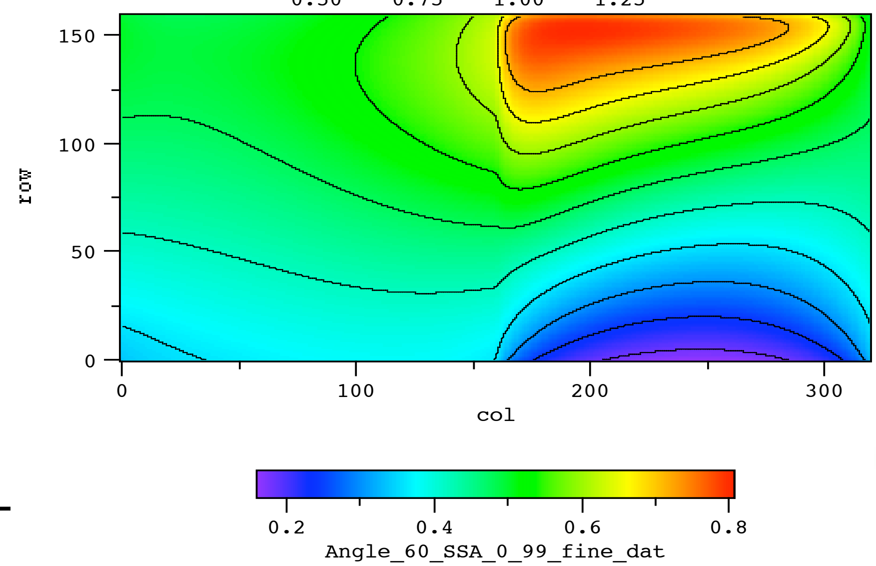
I3RC "Case 1" square-wave cloud: 0.25 km thick, alternating $\tau = 2{,}18$ every 0.25 km.



Diffuse photon densities

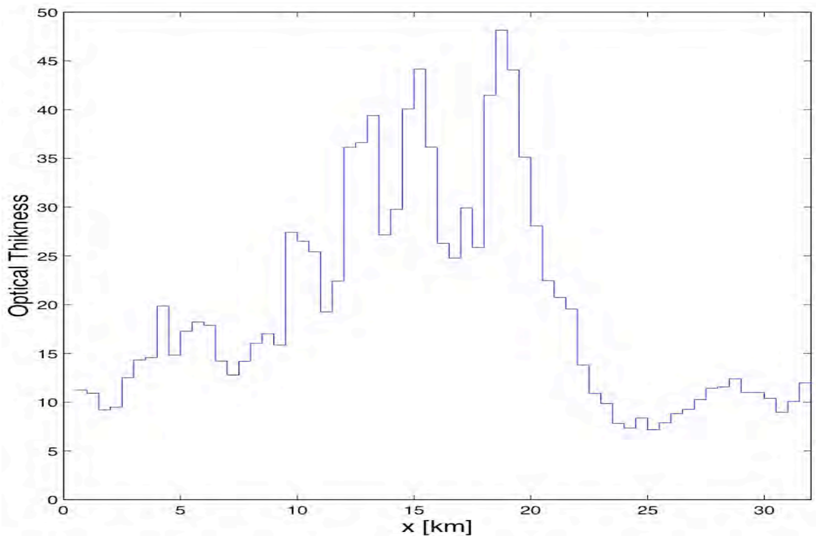
Direct flux (source term)



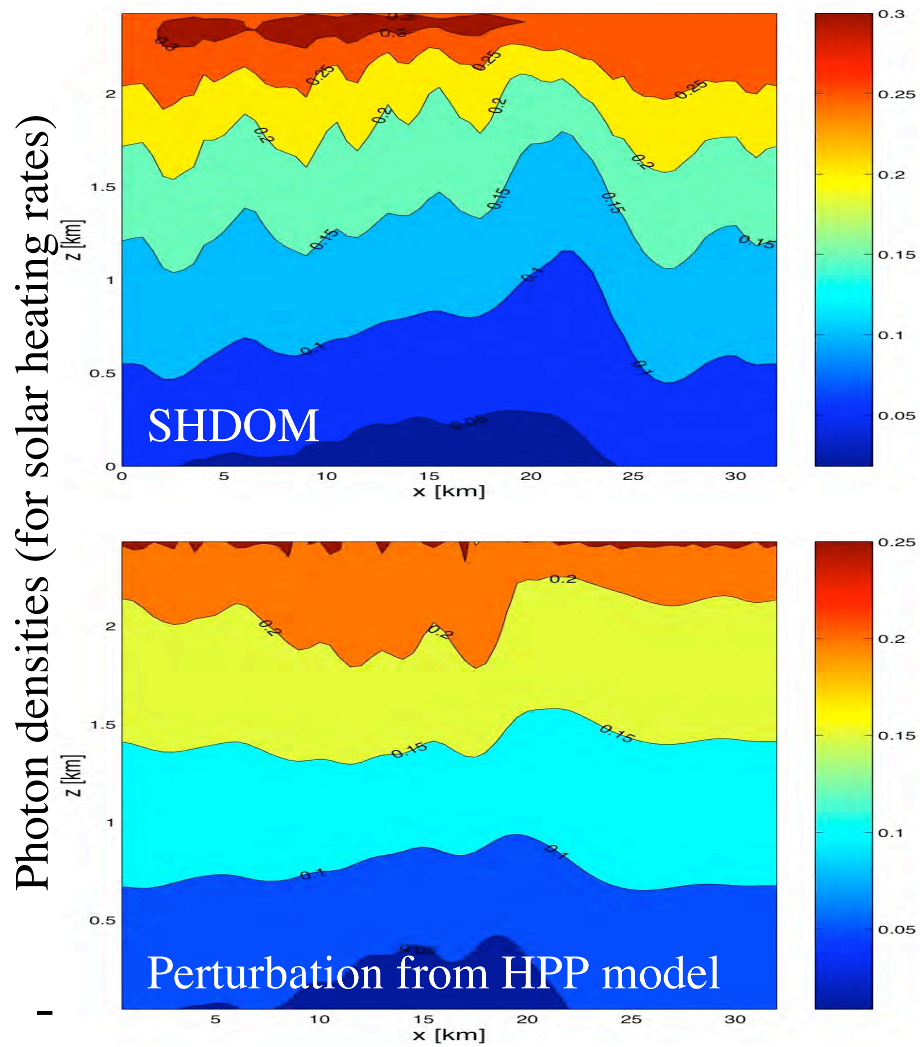


3D RT in Closed-Form Expressions: Adjoint Perturbation Theory

I3RC "Case 2" Cloud from MMCR+



- **Joint work with <u>Igor Polonsky (now at CSU)</u>**
- **Uses spatial Green functions for Homogeneous Plane-Parallel (HPP) clouds**
- **Perturbation could depart from**
  - ICA, rather than HPP model (done)
  - computational 3D diffusion model (cf. previous slide)



ACT 2

Passive from above, active from below

VOL. 54, NO. 2 JOURNAL OF THE ATMOSPHERIC SCIENCES 15 JANUARY 1997

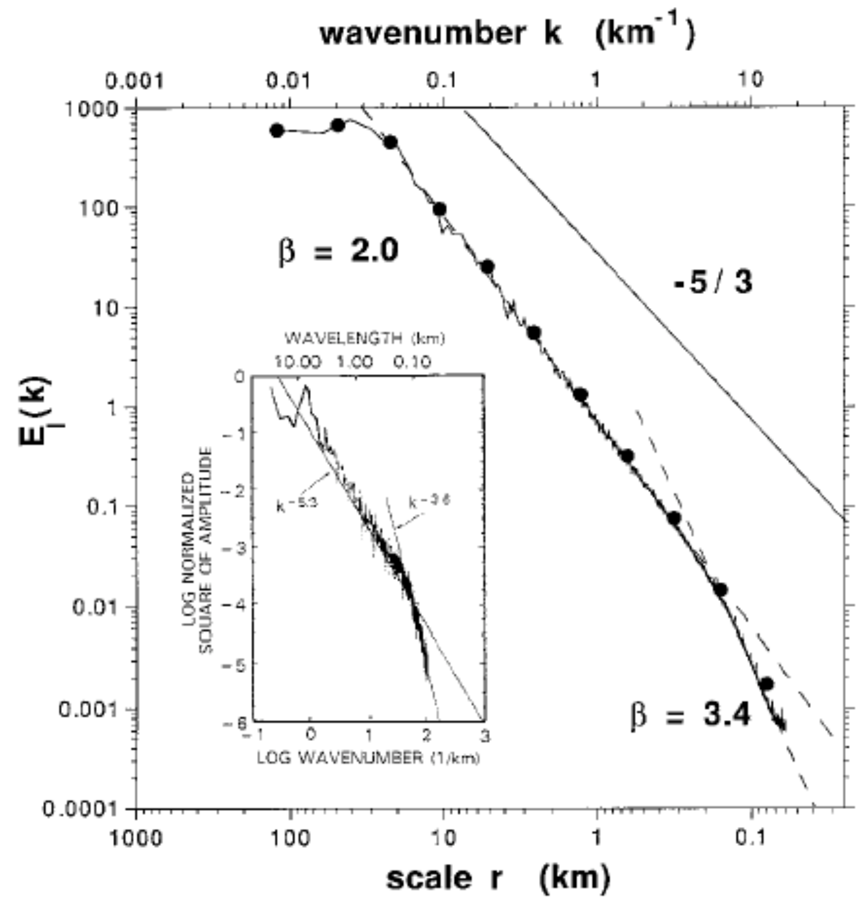
Context, c. 1995

The Landsat Scale Break in Stratocumulus as a Three-Dimensional Radiative Transfer Effect: Implications for Cloud Remote Sensing

ANTHONY DAVIS,* ALEXANDER MARSHAK,* ROBERT CAHALAN, AND WARREN WISCOMBE

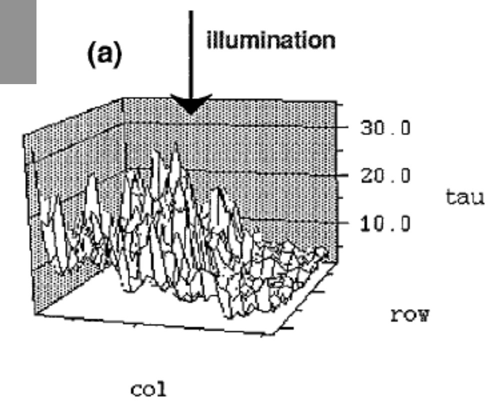
Climate and Radiation Branch, NASA/Goddard Space Flight Center, Greenbelt, Maryland

(Manuscript received 28 August 1995, in final form 16 May 1996)

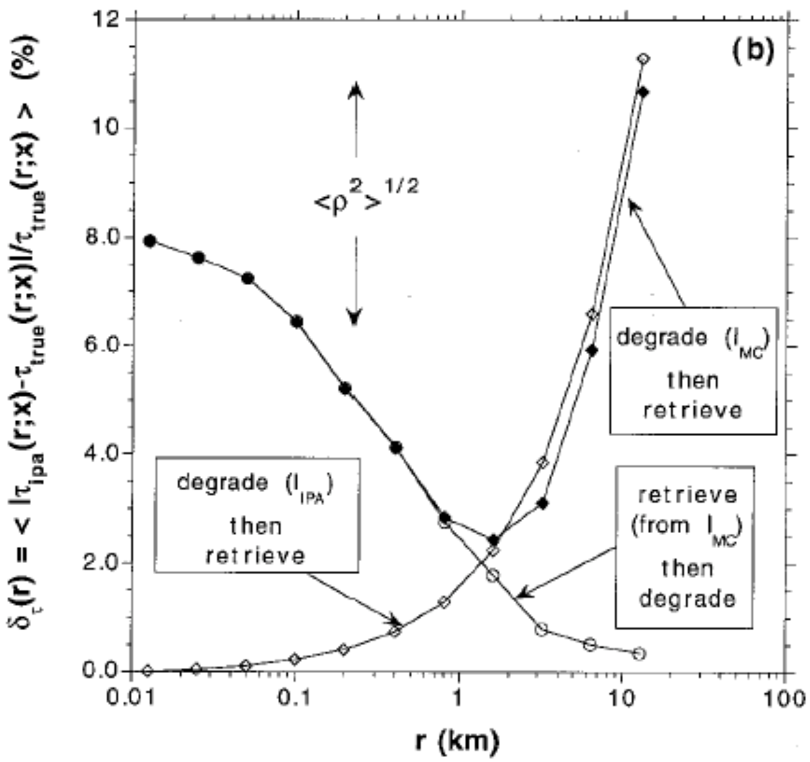


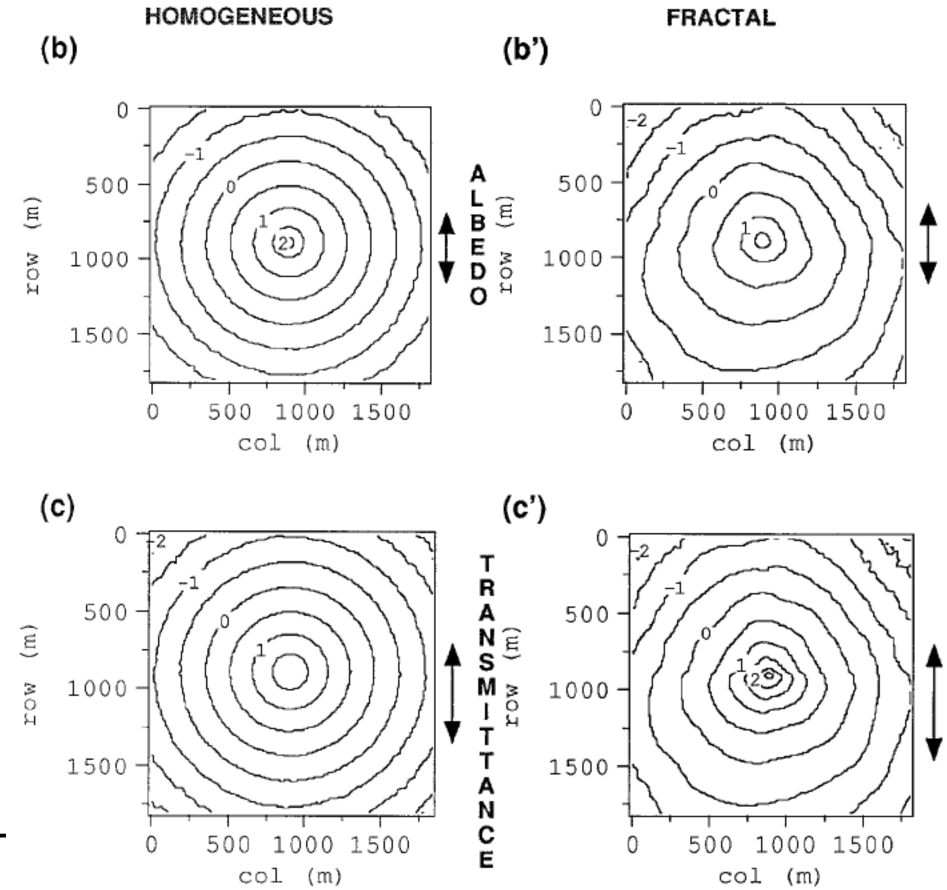
Context, c. 1995

“Sasha, trust me!”

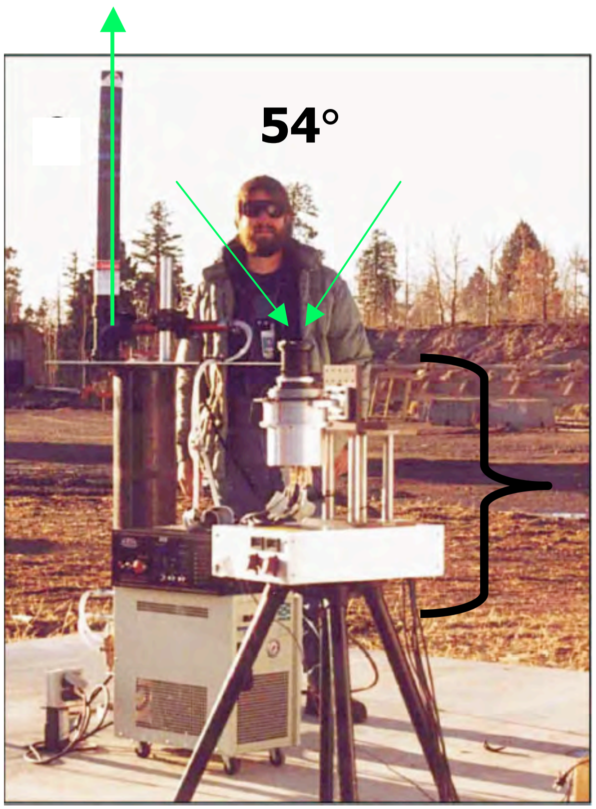


“No (pixel) scale left behind!”

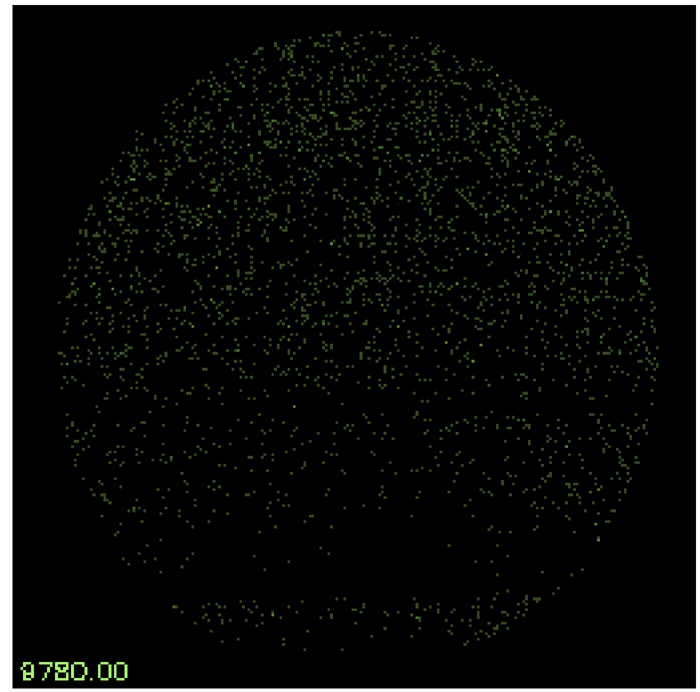


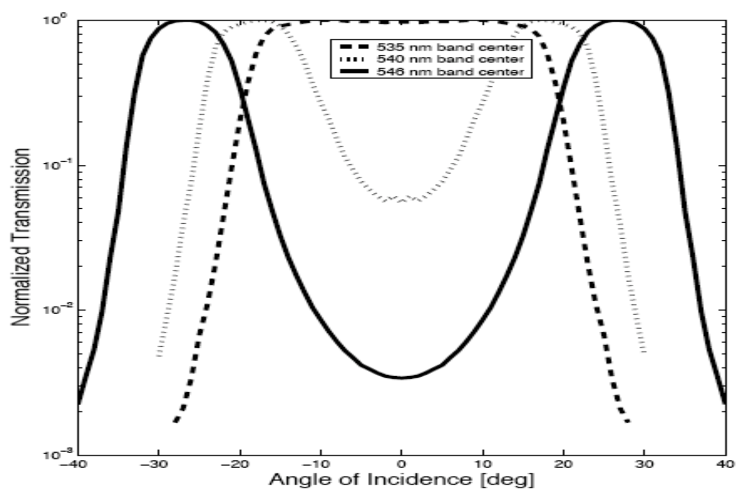


Wide-Angle Imaging Lidar (WAIL)

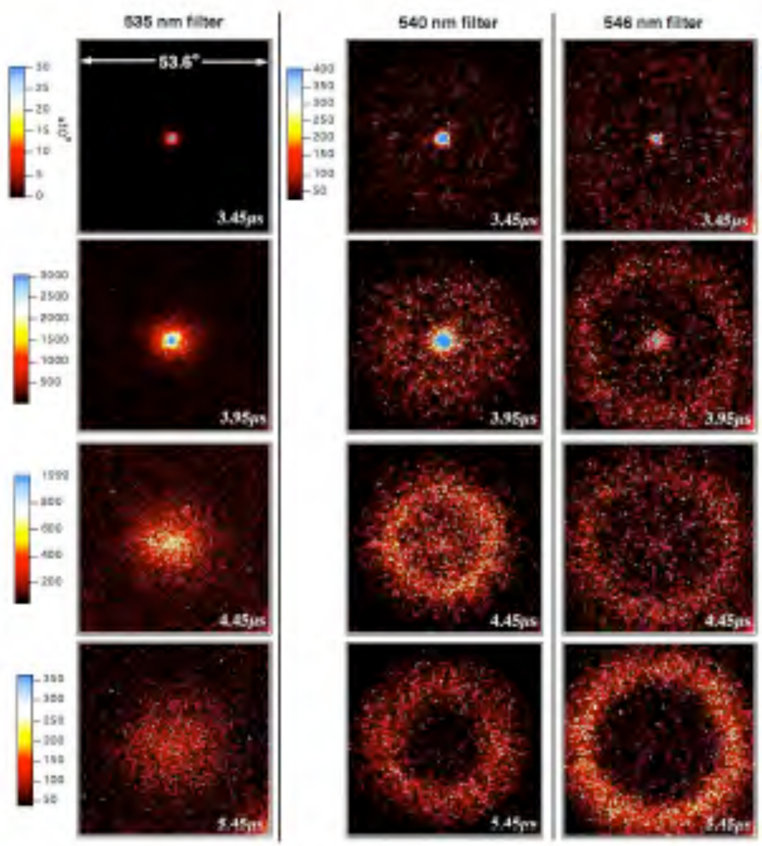


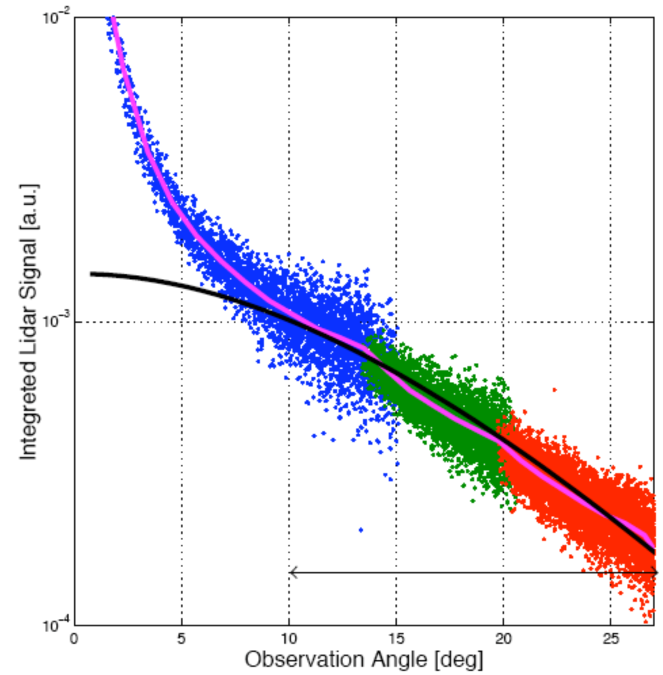
Remote Ultra Low
Light Imaging
(RULLI),
a specialty detector
by LANL/ISR-2

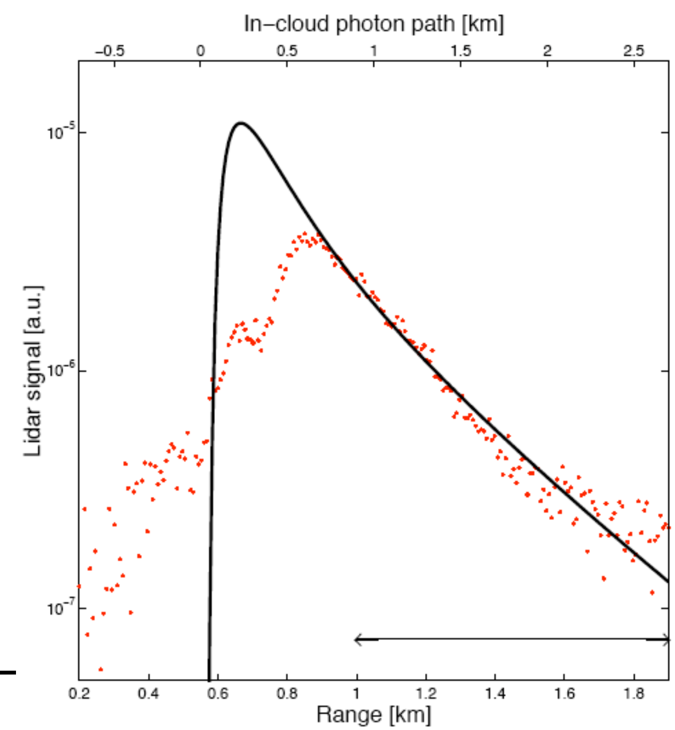




Wide-Angle Imaging Lidar (WAIL-2): New detector (gated/intensified CCD)

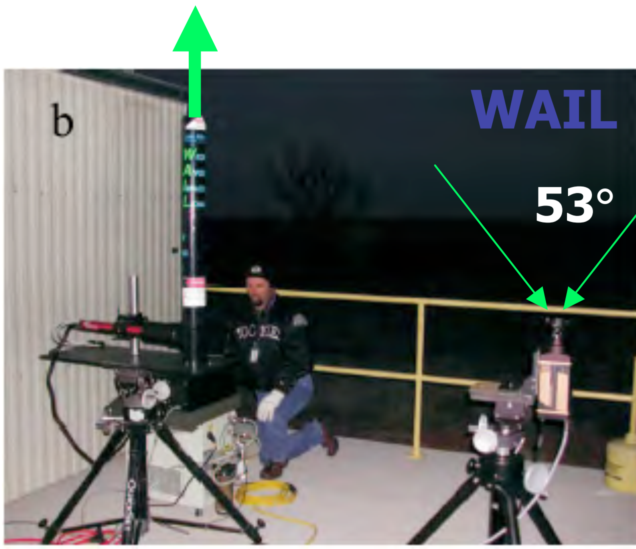






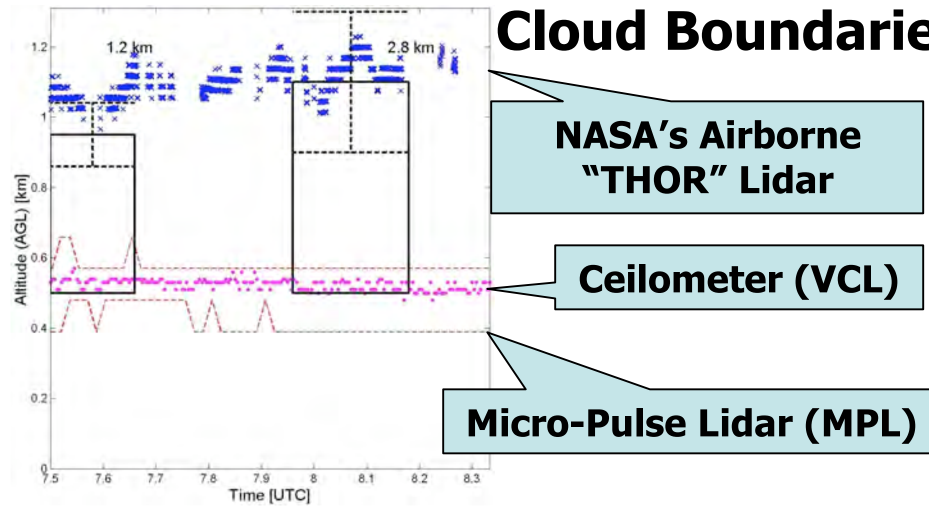
Wide-Angle Imaging Lidar (WAIL) validation campaign at ARM SGP site

Polonsky, Davis, Love

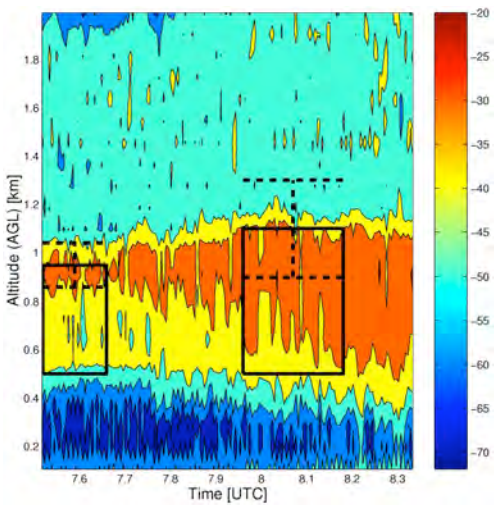


… captures and analyses the space-time Green function, in green light!

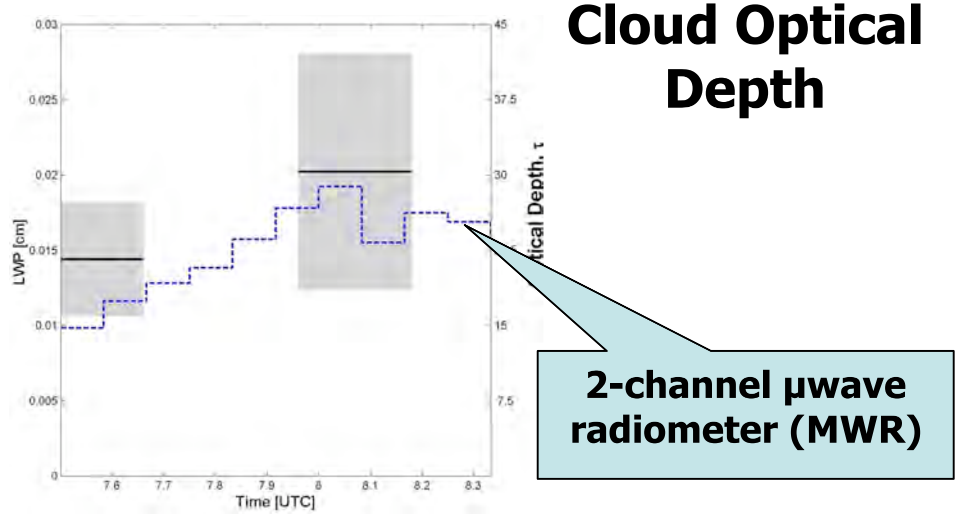
Cloud Boundaries



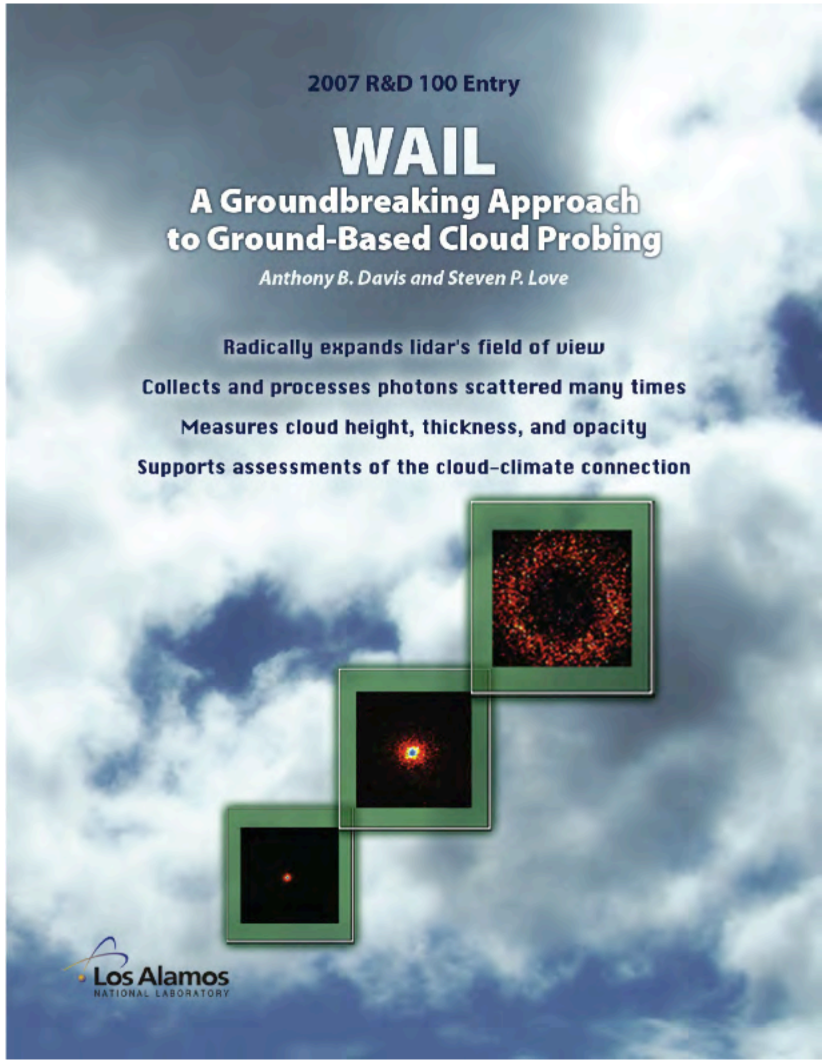
mm-Radar Reflectivity (MMCR)



Cloud Optical Depth



Wish us luck …

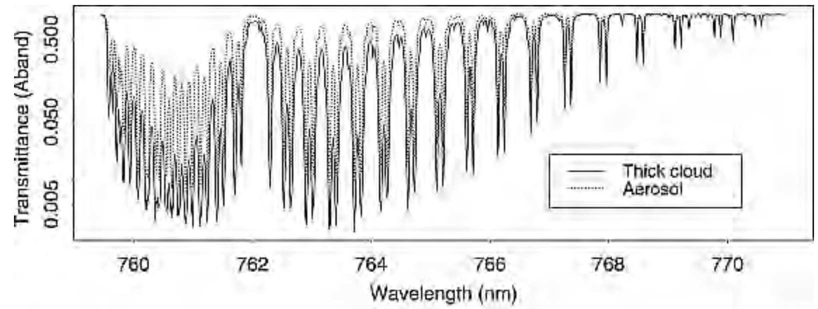


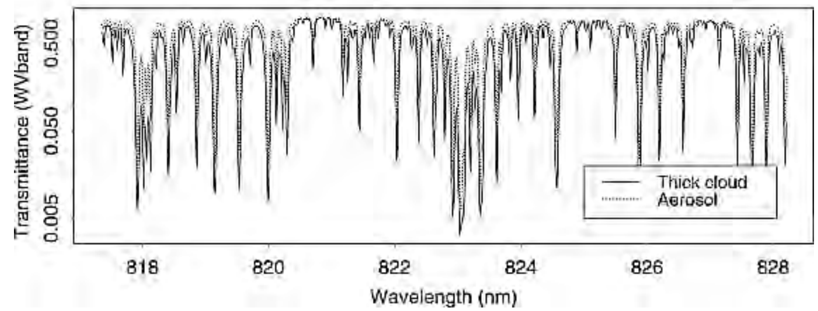
2007 R&D 100 Entry
WAIL
A Groundbreaking Approach to Ground-Based Cloud Probing
Anthony B. Davis and Steven P. Love
Radically expands lidar's field of view
Collects and processes photons scattered many times
Measures cloud height, thickness, and opacity
Supports assessments of the cloud-climate connection
Los Alamos
NATIONAL LABORATORY

ACT 3

RT in 3+1 dimensions, using a steady source

Differential absorption spectroscopy at high resolution





<u>From:</u> Min Q.-L., L. C. Harrison, P. Kiedron, J. Berndt, and E. Joseph, 2004: A high-resolution oxygen A-band and water vapor band spectrometer, *J. Geophys. Res.*, **109**, D02202, doi:10.1029/2003JD003540.

x-section **density** **pathlength**

$$I(\nu)/I_0 = \exp[-\sigma_\nu \times n \times L]$$

$$\text{known/not}: \begin{cases} ? & \surd & \surd & \text{estimating molecular cross-sections in the laboratory} \\ \surd & ? & \surd & \text{monitoring amounts of chemical effluent in situ} \\ \surd & \surd & ? & \text{scattering/reflection diagnostics of media permeated with gas} \end{cases}$$

$$I(\nu) = I(k_\nu) = \boxed{I_0} \int_0^\infty \exp[-k_\nu L] \boxed{p(L)}\, \mathrm{d}L \ \text{(equivalence "theorem")} \Rightarrow \langle L \rangle = -\left(\frac{\mathrm{d}}{\mathrm{d}k_\nu}\right) \ln I(k_\nu)$$

Temporal Green function of “3+1 D” RTE

Time-dependent <u>diffusion theory</u> for transmitted fluxes: Plane-parallel cloud model

H: cloud thickness

τ: cloud optical thickness ($\sigma H = H$ / mean-free-path)

g: asymmetry factor of scattering phase function

(i.e, $\langle\cos\theta_s\rangle = 2\pi\int \cos\theta_s p(\cos\theta_s) \sin\theta_s d\theta_s = 0.75 - 0.85$)

From asymptotic theory:

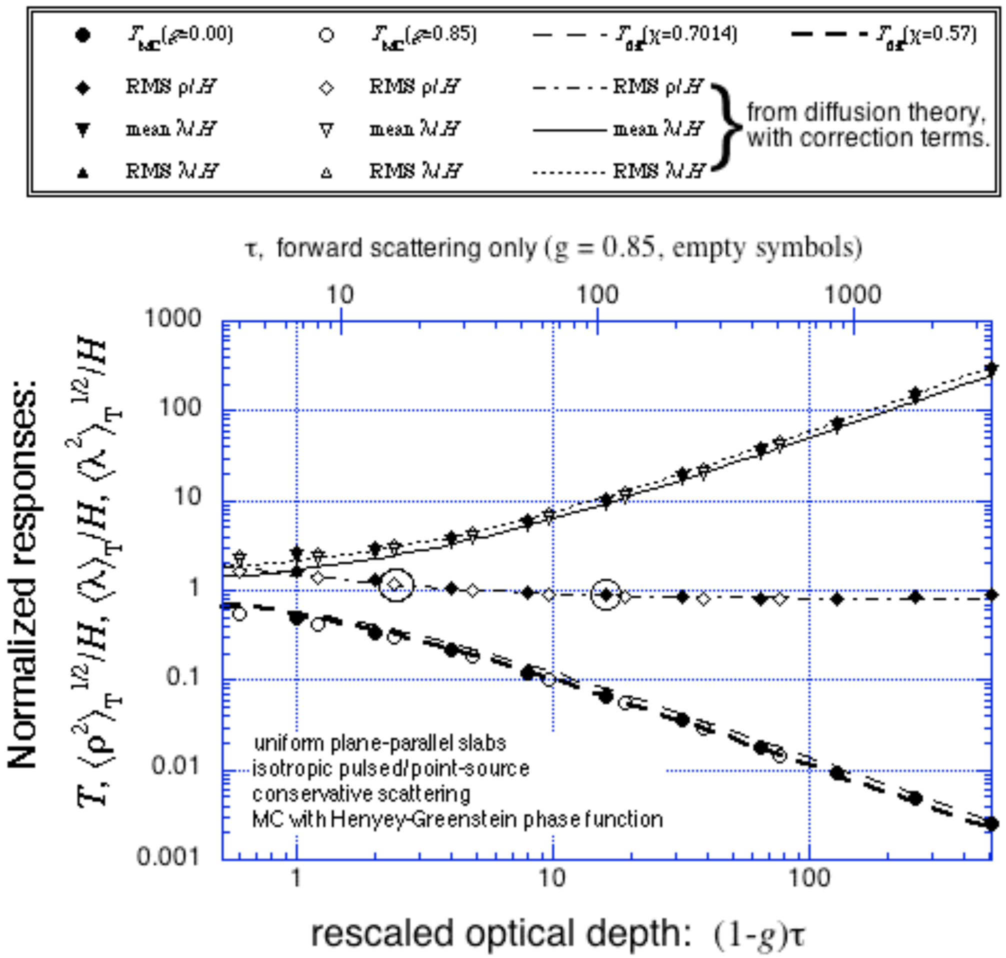
(i.e., scaling arguments based on random walk statistics)

$$\langle L \rangle_T \propto (1-g)\tau \times H$$

$$\langle \rho^2 \rangle_T^{1/2} \propto H^2$$

$$T \propto \frac{1}{(1-g)\tau}$$

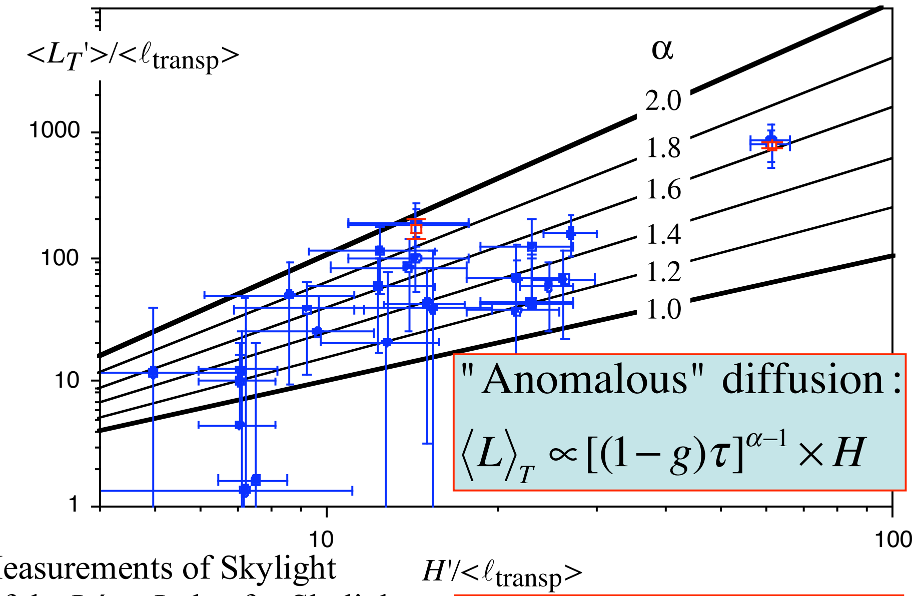
Exact diffusion theory vs. Monte Carlo?



$$\langle L^2 \rangle_T^{1/2} / \langle L \rangle_T \approx \sqrt{7/5} \approx 1.18$$

Davis, A. B., and A. Marshak, 2002: Space-time characteristics of light transmitted by dense clouds, A Green function analysis, *J. Atmos. Sci.*, **59**, 2713-2727.

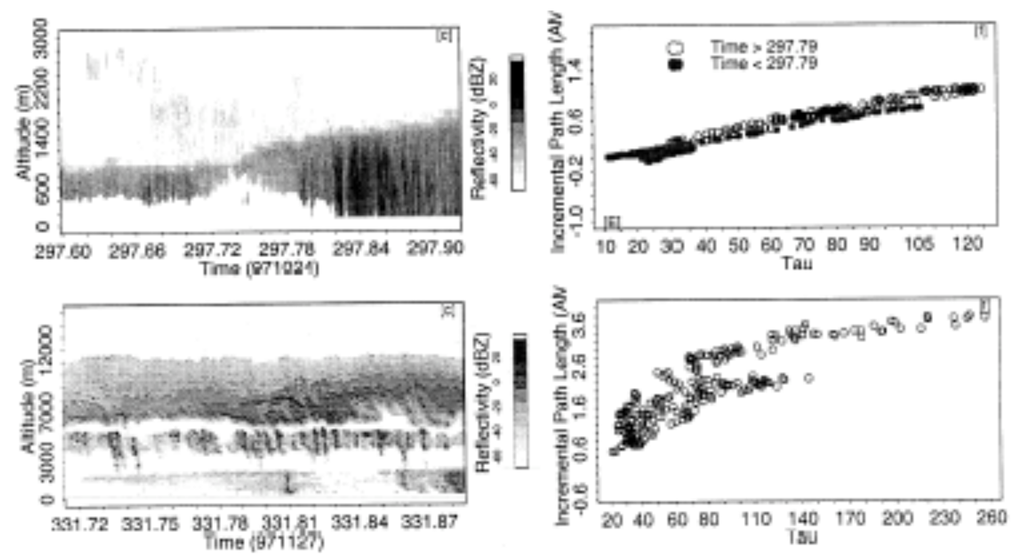
Ground-based oxygen spectroscopy



(Davis & Marshak, 1997)

Cases near the α=2 line are very overcast, and those near α=1 are for sparse clouds, as expected from model.

Pfeilsticker, K., 1999: First Geometrical Pathlength Distribution Measurements of Skylight Using the Oxygen A-band Absorption Technique - II, Derivation of the Lévy-Index for Skylight Transmitted by Mid-Latitude Clouds, *J. Geophys. Res.*, **104**, 4101-4116.



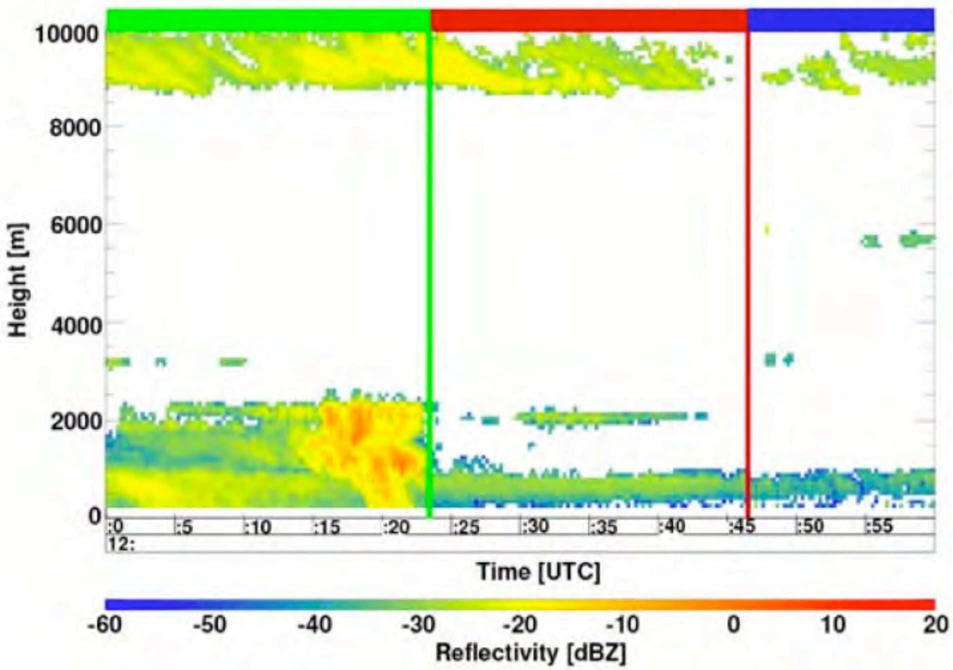
A single cloud layer (α=2) with variable thickness $H \propto$ the slope of the linear path vs optical depth plot.

A complex cloud situation (1<α<2) with multi-layers, some broken; power-laws in α–1 will fit the data.

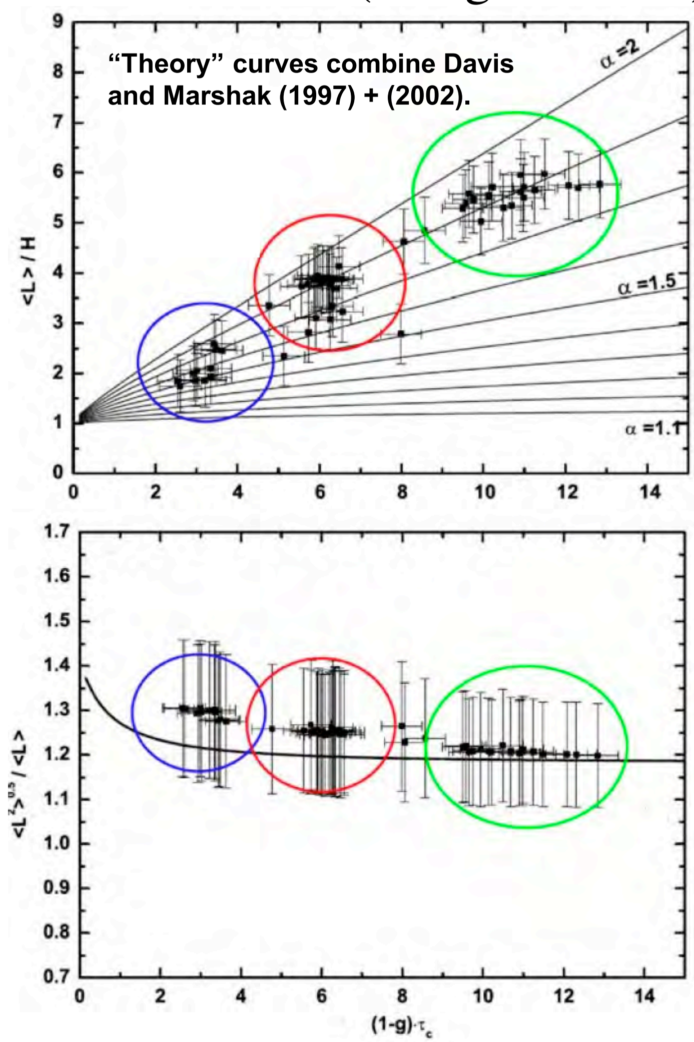
Min, Q.-L., L. C. Harrison, and E. E. Clothiaux, 2001: Joint statistics of photon path length and cloud optical depth: Case studies, *J. Geophys. Res.*, **106**, 7375-7385.

From 1st to 2nd Moments …

KNMI 35 GHz cloud radar

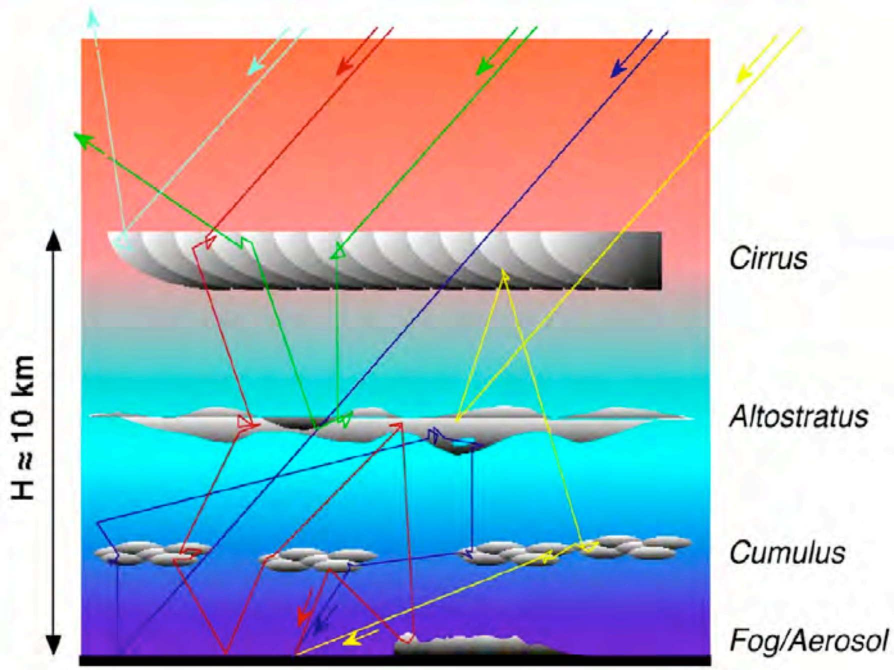


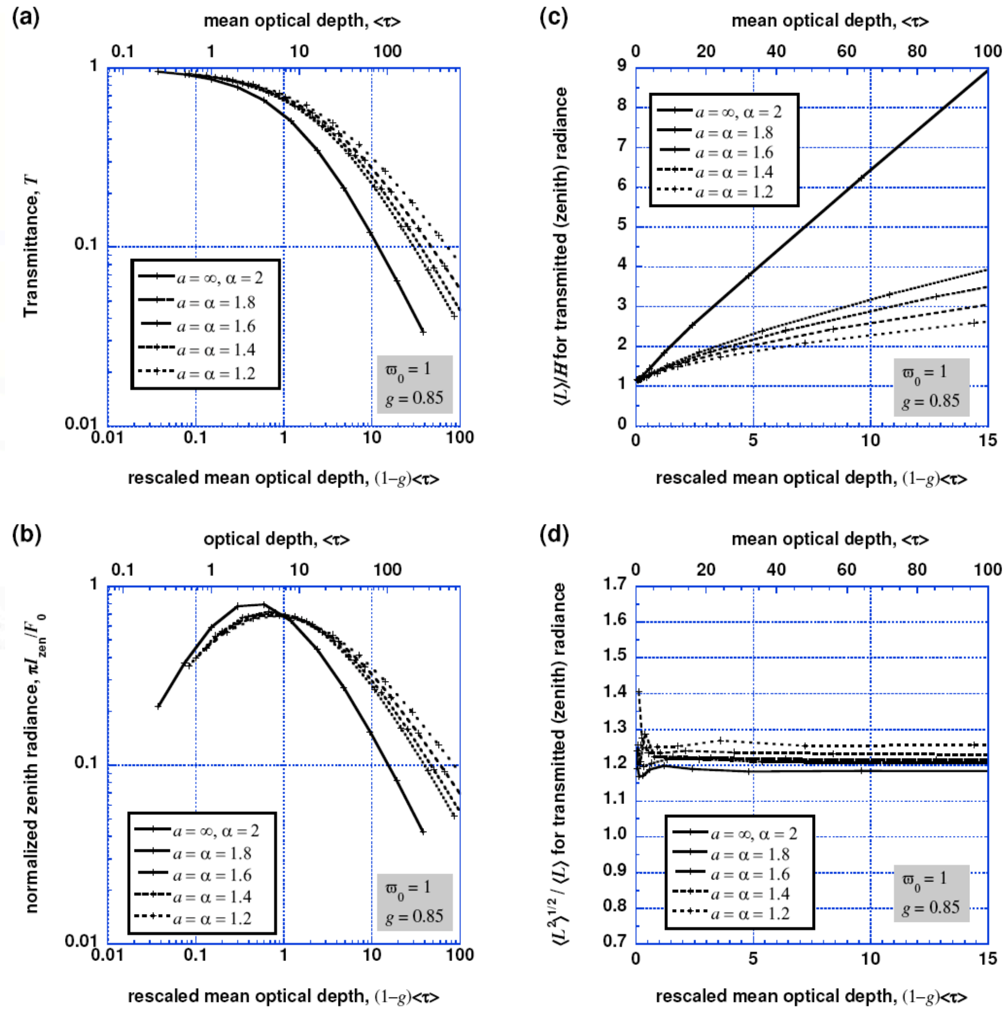
U. Heidelberg O_2 A-band spectrometer (2nd generation)



Scholl, T., K. Pfeilsticker, A. B. Davis, H. Klein Baltink, S. Crewell, U. Löhnert, C. Simmer, J. Meywerk, and M. Quante, 2006:
Path Length Distributions for Solar Photons Under Cloudy Skies: Comparison of Measured First and Second Moments with Predictions from Classical and Anomalous Diffusion Theories,
J. Geophys. Res., *vol. 111*, D12211-12226.

Anomalous *Transport* Model





Solve 1D integral RTE with

$$T(s) = \mathrm{e}^{-\sigma s} \mapsto$$

$$\langle T_a(s) \rangle = \frac{1}{\left(1 - \frac{\langle \sigma \rangle s}{a}\right)^a}, 1 < a \le \infty$$

Davis, A. B., 2006: Effective Propagation Kernels in Structured Media with Broad Spatial Correlations, Illustration with Large-Scale Transport of Solar Photons Through Cloudy Atmospheres, in *Computational Methods in Transport – Granlibakken 2004* (Lecture Notes in Computational Science and Engineering), F. Graziani (Ed.), Springer-Verlag, New York (NY), pp. 85-140.

1. <u>Build</u> GCM SW transport schemes (1D-type RT models for averages over large domains)

- **Conceptualize <u>unresolved</u> variability**
  - Cloud fraction, cloud aspect ratio
  - Internal variability?
  - Spatial correlations, including layer-to-layer overlap
  - Need other parameters?
- **Make reasonable statistical assumptions**
  - Fractals and power-laws
  - Exponentials, lognormals, Gamma, etc.
- **Use judicious approximations in the RT**
- **Write code … and *verify* it!**

2. Test GCM SW transport schemes with detailed 3D RT

- **Need 3D clouds from observations**
  - Could use CSRM output [OK for starts]
  - Better to use imagers and profilers
    - Ground-based: ARM sites
    - Space-based: A-train, especially CloudSat/Calipso
  - Best to use ARM Volume-imaging Array (AVA)
    - Need confidence in extracting optical properties
- **Need *verified* 3D RT codes**
  - full cloudy column capability
  - use to “assimilate” available data
- **Compare 3D (spatially averaged) and 1D-type RT model outputs for given inputs**

3. Validation of GCM grid-scale SW transport schemes, a.k.a. “closure” experiments

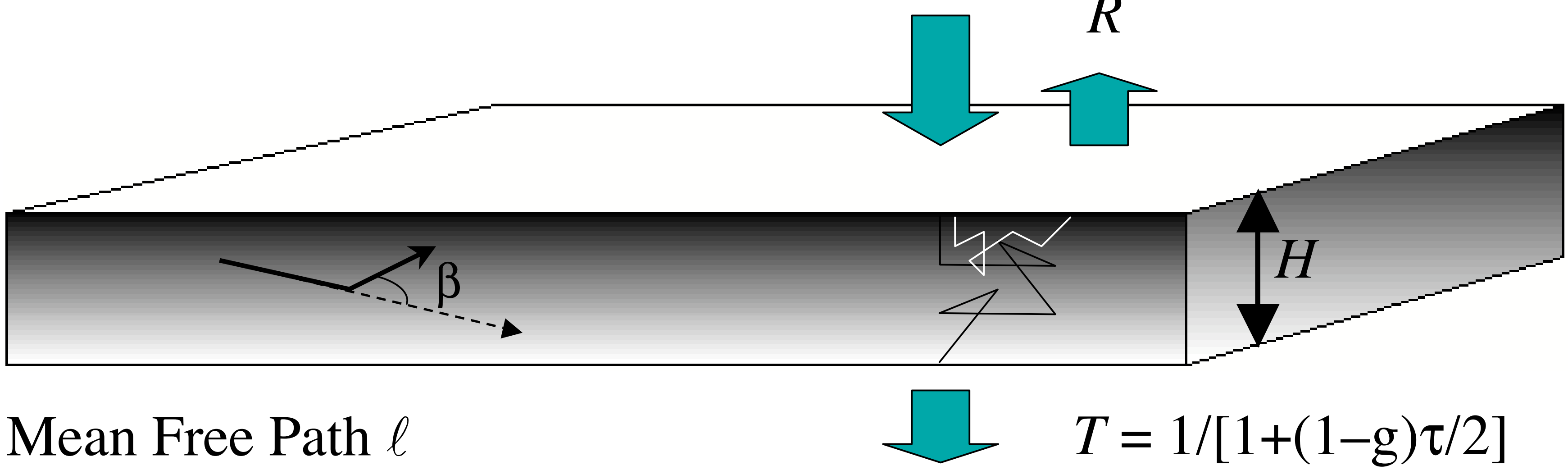
- **Need new observations directly related to the domain-average HR profile predictions (i.e., output of new/improved SW transport codes)**
  … at selected λ’s
- **Focus on process (physics) of absorption by interstitial gases under complex cloudy conditions**
  - **use O_2 A-band:** known amount and known cross-section, but unknown path distribution
  - from below (ARM), within (UAVs), and above (OCO) …
  - fully stress the 1D-type RT model: use scenarios with multiple and/or broken cloud layers

ACT 4

Seeing clouds from both sides …

Passive 3D cloud remote sensing in VNIR, 1

Take wavelength where there is no absorption,
hence R(eflectance) + T(ransmittance) = 1



Mean Free Path ℓ

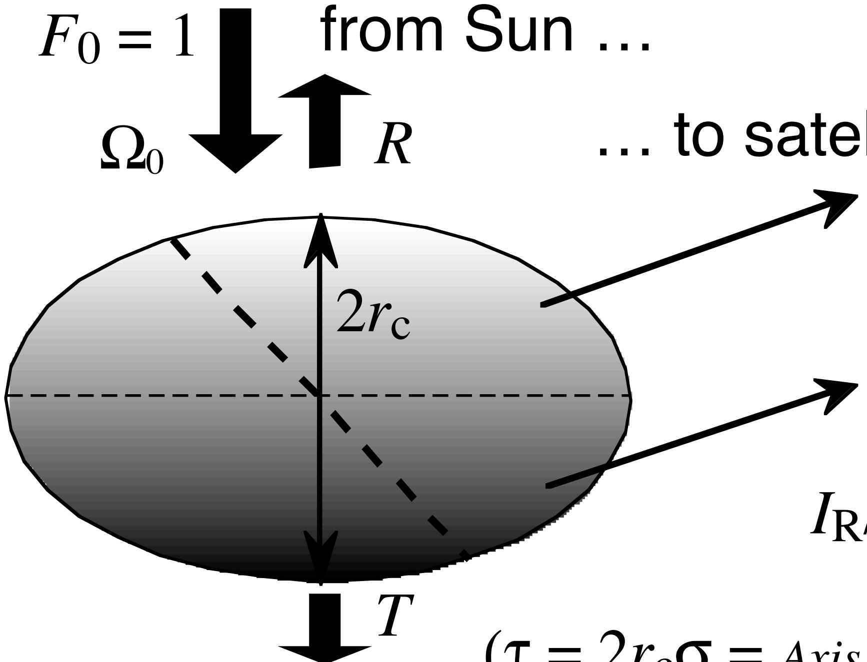
$T = 1/[1+(1-g)\tau/2]$

"Asymmetry" factor $g = \langle\cos\beta\rangle \approx 0.85$

"Transport" MFP $\ell_t = \ell/(1-g)$
"Optical" Depth $\tau = H/\ell \gg 1$
"Rescaled" OD $= H/\ell_t = (1-g)\tau$

Schuster, A., 1905: Radiation through a foggy atmosphere, *Astrophys. J.*, **21**, 1-22.

Passive 3D cloud remote sensing in VNIR, 2



$I_R \approx R \times F_0/\pi$

$I_T \approx T \times F_0/\pi$

$$I_R/I_T \approx R/T = (1-g)\tau/2\chi$$

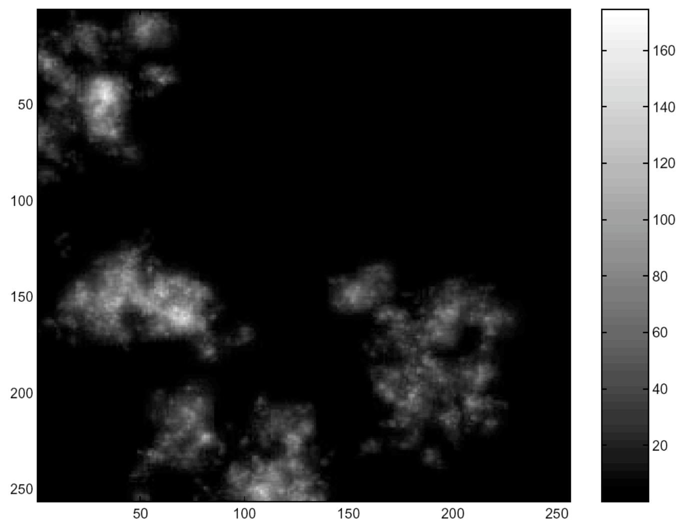
$\surd$ **?** $\surd$

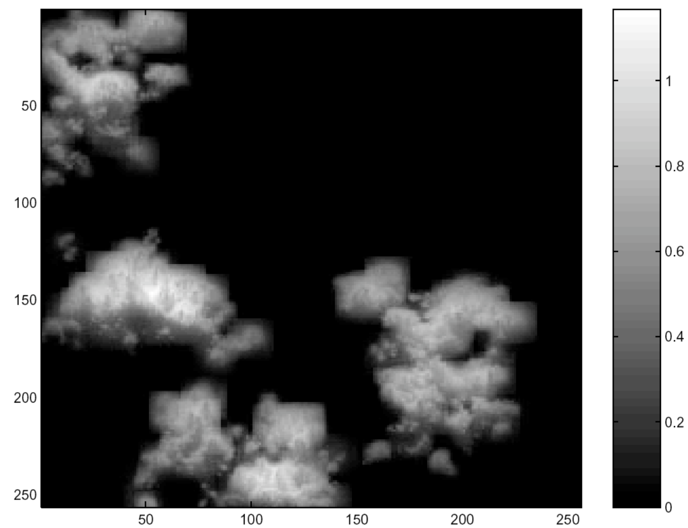
($\tau = 2r_c\sigma$ = *Axis along* Ω_0/*Mean-Free-Path*)

Davis, A. B., 2002: Cloud remote sensing with sideways-looks: Theory and first results using Multispectral Thermal Imager (MTI) data, in *S.P.I.E. Proceedings*, *vol. 4725: "Algorithms and Technologies for Multispectral, Hyperspectral, and Ultraspectral Imagery VIII,"* Eds. S. S. Shen and P. E. Lewis, S.P.I.E. Publications, Bellingham (Wa), pp. 397-405.

Unpublished generalizations with **Mathematica** and with **Igor Polonsky** (LANL PDF, now CSU).

Passive 3D cloud remote sensing in VNIR, 3

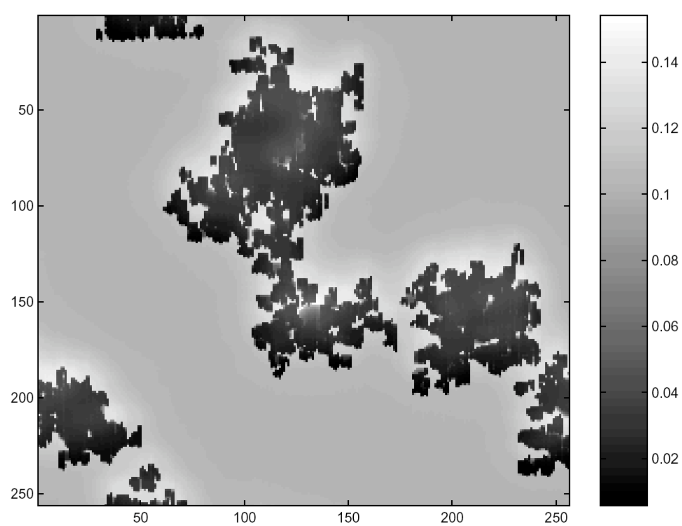




Will be used in validation of new τ retrieval scheme (Polonsky et al., in preparation)

Cloud Adjacency Effects, Simulated

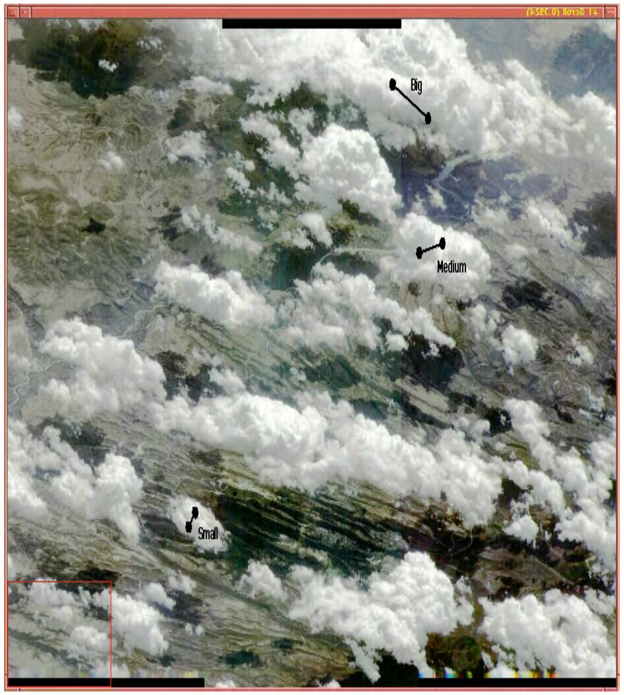
- 3D clouds by wavelet-like “tdMAP” model (Benassi et al., 1999, 2004)
- 3D radiances by SHDOM code (Evans, 1998)



- Surface albedo assumed uniform at 0.1
- Appear to be as much as 50% more near clouds
- Depends on wavelength, hence confusion in spectral matching

Passive 3D cloud remote sensing in VNIR, 4

- ≈100 years after Schuster (1905), ≈50 after Chandrasekhar-Wick DOM in 1D, and ≈25 after 1D asymptotic theory … a simple 3D RT solution for cloud remote sensing. **Three applications areas:**
  - Hi-res imagery
    - Landsat, ASTER, MTI (DOE/NNSA)
    - ARM's Whole Sky Imagers?
  - Cloud-capable atmospheric compensation for remote sensing of surface properties
  - Effects of unresolved structure



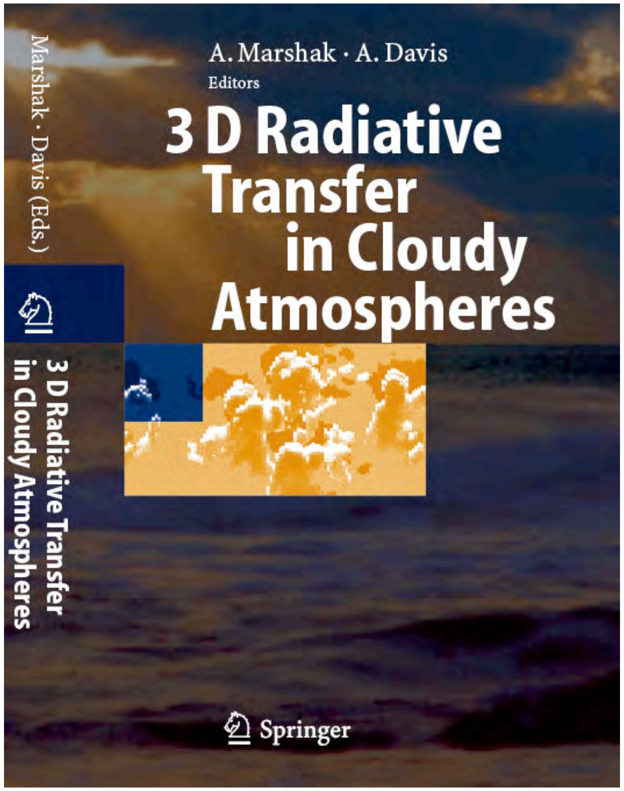
Epilogue

Research Capabilities

- **Atmospheric radiative transfer in or in the presence of clouds (i.e., dense structured media)**
  - Three-dimensional, for diagnostics & for energetics
  - Time-dependent, as needed
  - Theoretical (computational + analytical) and observational
- **Remote sensing techniques**
  - Passive and active, mostly in the optical (solar) spectrum
  - Physics-based *multi-pixel* exploitation methods
  - Instrument development
- **Concepts from nonlinear geophysics**
  - Multi-scale (wavelets, fractals, multifractals, etc.)
  - Stochastic modeling (cloud structure, etc.)
  - Model calibration, verification & (process of) validation

Questions?

… for *some* answers:



(appeared in 2005)

RT as a Linear Boltzmann Problem

- $I_\lambda(ct,\boldsymbol{x},\Omega)$ is "radiance" ~ $f(t,\boldsymbol{x},\boldsymbol{p})$ for Boltzmann's mesoscopic equation
  - t is time and ct is total path (but we'll consider stationary sources here)
  - $\boldsymbol{x}$ is position
  - Ω is direction of propagation (from $\boldsymbol{p}/p$)
  - λ is wavelength (from $h/p = ch/E$, $E = h\nu$)
- $\sigma(\boldsymbol{x})$ is the given "extinction" field = cross-section × (fluctuating) density
  - $\sigma(\boldsymbol{x})$ is the collision probability per unit of length along a beam (we assume geometric optics)
  - $\sigma_s(\boldsymbol{x}) \leq \sigma(\boldsymbol{x})$ is the collision probability per unit of length for a scattering (rather than an absorption)
  - $\boldsymbol{x} \in \mathrm{M} \subseteq \mathrm{R}^3$, the optical medium
- $f_v(\boldsymbol{x})$, $\boldsymbol{x} \in \mathrm{M}$, and $f_s(\boldsymbol{x})$, $\boldsymbol{x} \in \partial\mathrm{M}$, are the bulk and boundary emission rates.

$$I(\boldsymbol{x},\Omega) = \int_0^{s_b(\boldsymbol{x},-\Omega)} \sigma_s(\boldsymbol{x}-\Omega s')\mathrm{e}^{-\int_0^{s'}\sigma(\boldsymbol{x}-\Omega s'')\mathrm{d}s''}\left[\int_{4\pi} p(\boldsymbol{x}-\Omega s',\Omega'\to\Omega)I(\boldsymbol{x}-\Omega s',\Omega')\mathrm{d}\Omega'\right]\mathrm{d}s' + Q(\boldsymbol{x},\Omega)$$

with source term $Q(\boldsymbol{x},\Omega) = \int_0^{s_b(\boldsymbol{x},-\Omega)} f_v(\boldsymbol{x}-\Omega s')\mathrm{e}^{-\int_0^{s'}\sigma(\boldsymbol{x}-\Omega s'')\mathrm{d}s''}\mathrm{d}s' + f_s(\boldsymbol{x}-\Omega s_b(\boldsymbol{x},-\Omega))\mathrm{e}^{-\int_0^{s_b(\boldsymbol{x},-\Omega)}\sigma(\boldsymbol{x}-\Omega s')\mathrm{d}s'}$

"propagation" kernel: $T_0(s;\boldsymbol{x},\Omega) = \mathrm{e}^{-\int_0^{s}\sigma(\boldsymbol{x}-\Omega s')\mathrm{d}s'} = \exp[-\underbrace{\bar{\sigma}(s,\boldsymbol{x}+\Omega s/2)}_{\text{random}} \times \underbrace{s}_{\text{fixed}}]$

$$\boxed{\ldots\ \langle T_0(s)\rangle?}$$

When σ(*x*) is a function …

$$\overline{\sigma}(r,x) = \frac{1}{r}\int_{x-r/2}^{x+r/2} \sigma(x')\mathrm{d}x'$$

Empirical necessity to define a density ("1-point scale independence"):

$$\overline{\sigma}(r,x) \overset{\mathrm{d}}{=} \overline{\sigma}(r',x)$$

for r and r' in some large enough range.

$$\langle \sigma(x+r) - \sigma(x) \rangle \sim |r|^{h(x)}, 0 < h(x) \leq 1, \quad \Rightarrow \quad \lim_{r\to 0} \overline{\sigma}(r;x) = \sigma(x) \in \mathrm{R}^+, \forall x.$$

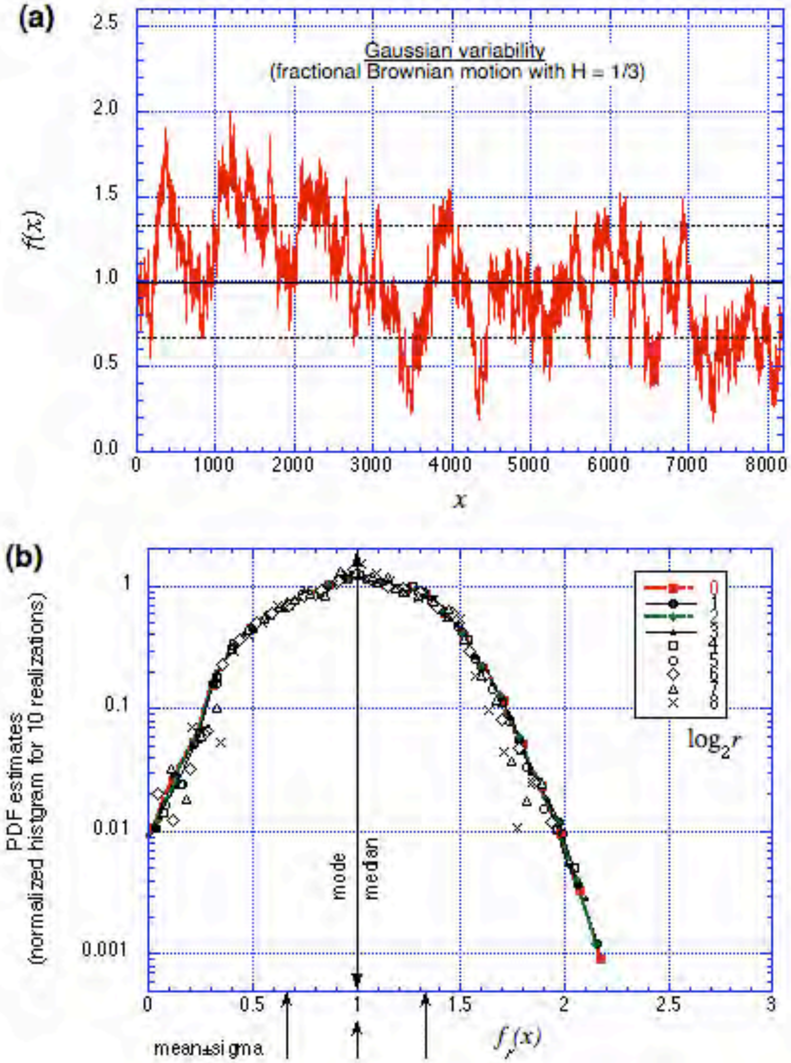
I. $\langle T_0(s)\rangle = \langle \Pr\{\text{step}>s\}\rangle$ is exponential if and only if $\boldsymbol{\sigma(x)} \equiv \mathit{const.}$

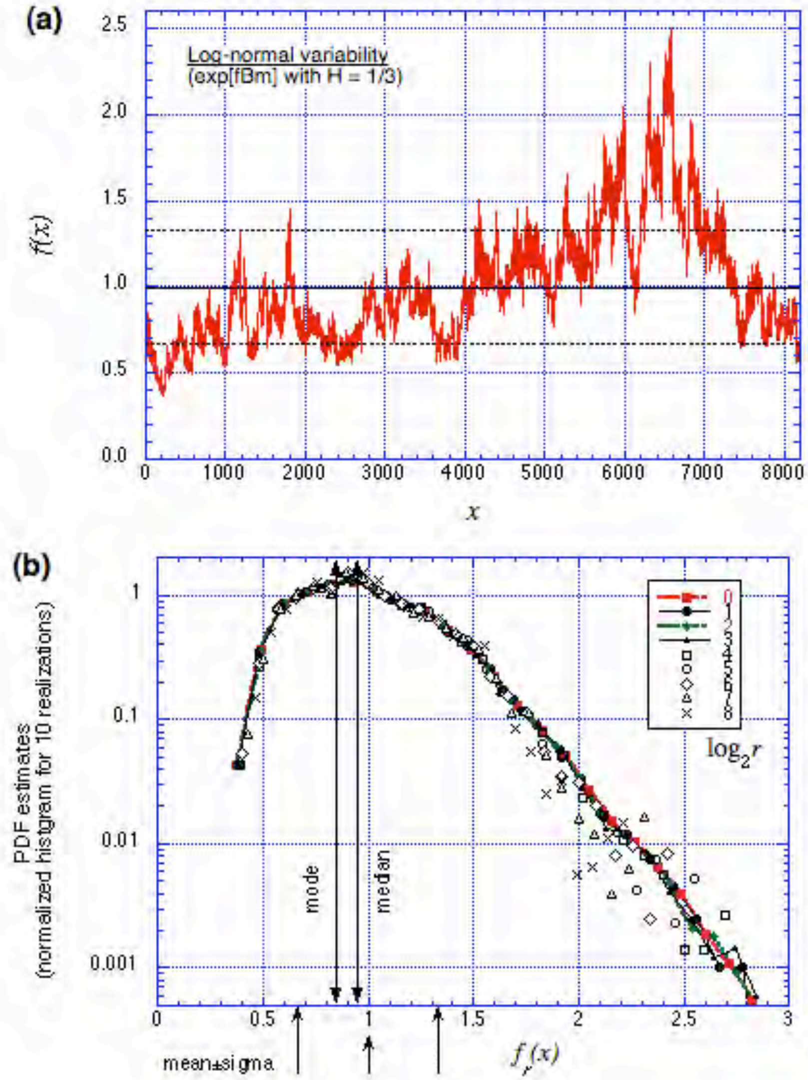
II. Mean-Free-Path (MFP) is minimal (and $=1/\sigma$) if $\boldsymbol{\sigma(x)} \equiv \mathit{const.}$

III. $\langle T_0(s)\rangle$ is sub-exponential, even if the real MFP is used (rather than $1/\langle\sigma\rangle$).

Davis, A. B., and A. Marshak, Photon propagation in heterogeneous optical media with spatial correlations: Enhanced mean-free-paths and wider-than-exponential free-path distributions, *J. Quant. Spectrosc. Rad. Transf.*, ***84***, 3-34 (2004).

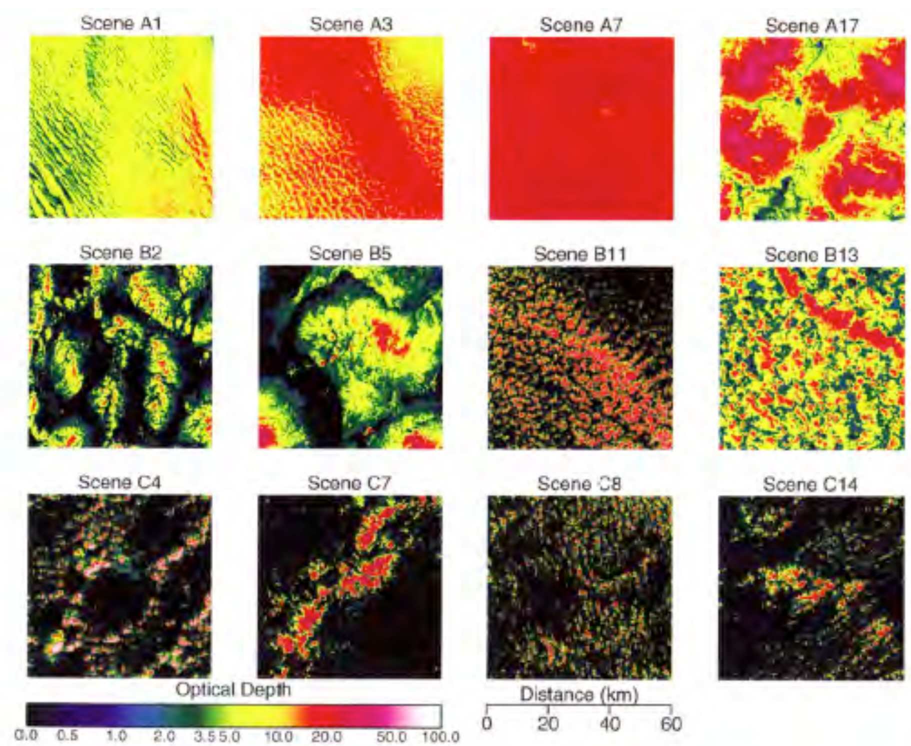
Synthetic scale-invariant media that are turbulence-like

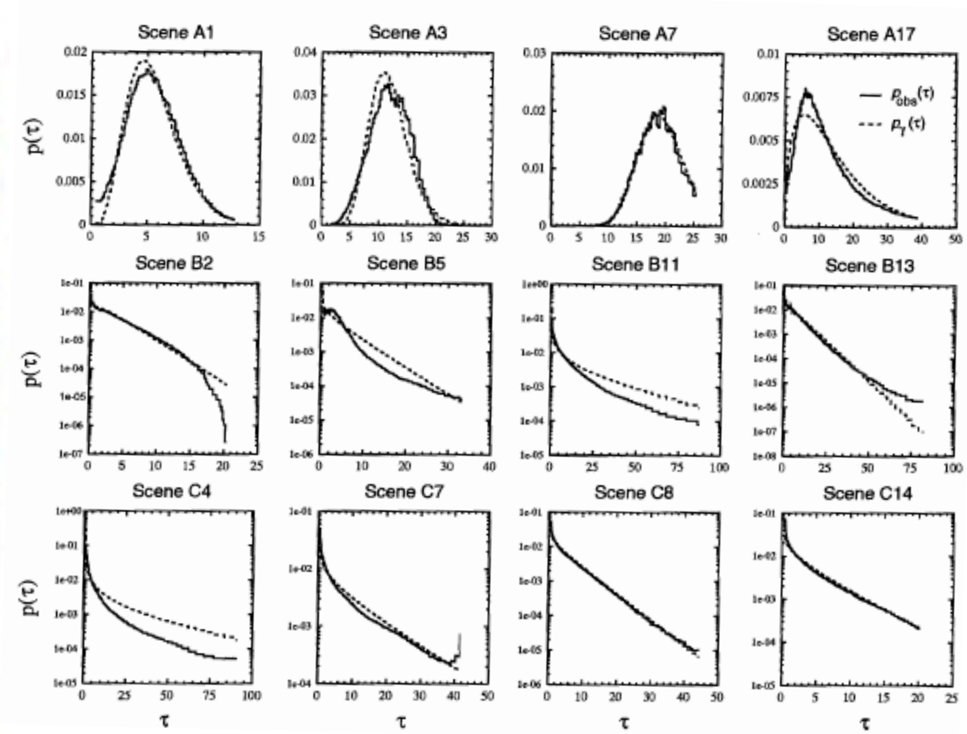




Expectations for Earth's cloudy atmosphere, 1: *Barker et al.'s (1996) LandSat Data Analysis*

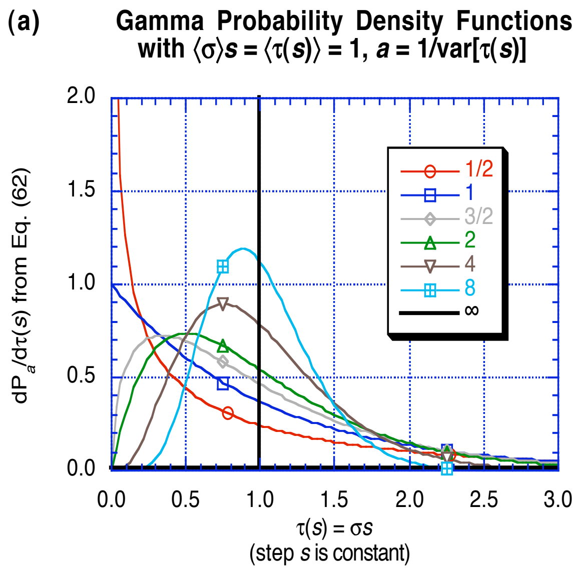
Gamma distributions capture many cloud optical depth scenarios.





From: Barker, H. W., B. A. Wielicki, and L. Parker, 1996: A parameterization for computing grid-averaged solar fluxes for inhomogeneous marine boundary layer clouds - Part 2, Validation using satellite data, *J. Atmos. Sci.*, **53**, 2304-2316.

Expectations for Earth’s cloudy atmosphere, 2: *Effective transport kernels are power-law!*

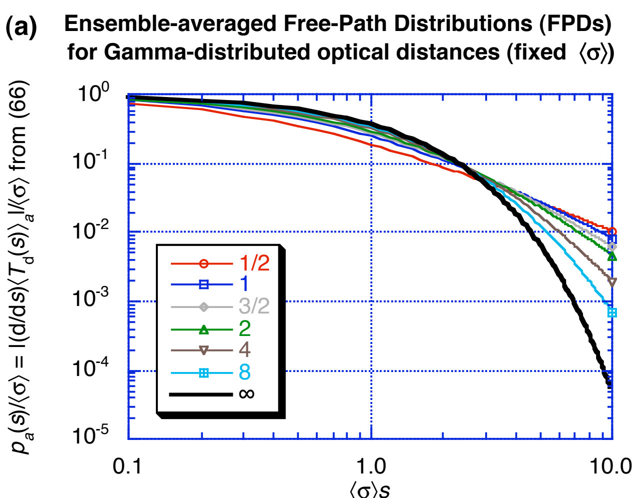


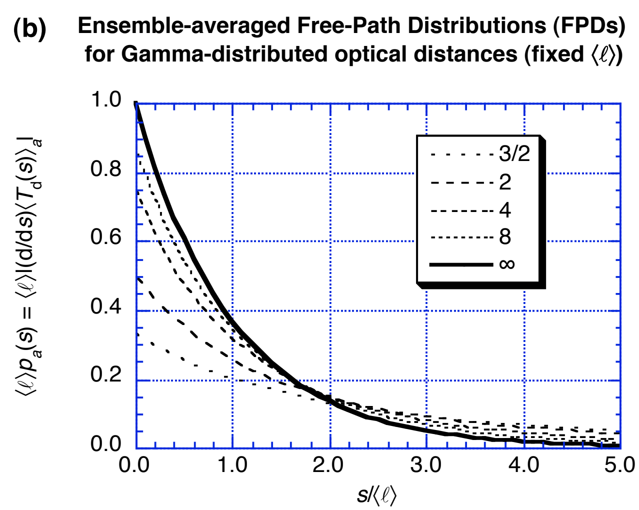
Assuming $s = H$ (thickness) in previous slide:

$$p(\bar{\sigma};\langle\bar{\sigma}\rangle,a)\approx\frac{1}{\Gamma(\nu)}\left(\frac{a}{\langle\bar{\sigma}\rangle}\right)^{a}\bar{\sigma}^{a-1}\exp[-a\frac{\bar{\sigma}}{\langle\bar{\sigma}\rangle}],$$

where $a=\dfrac{1}{\langle\bar{\sigma}^2\rangle/\langle\bar{\sigma}\rangle^2-1}$, yields mfp $\ell=\left\langle\dfrac{1}{\bar{\sigma}}\right\rangle=\dfrac{a}{a-1}\dfrac{1}{\langle\bar{\sigma}\rangle}$

and $\langle T_0(s)\rangle=\langle\exp[-\bar{\sigma}s]\rangle=\dfrac{1}{\left(1+\dfrac{\langle\bar{\sigma}\rangle s}{a}\right)^{a}}=\dfrac{1}{\left[1+\dfrac{s/\ell}{(a-1)}\right]^{a}}.$





NNSA

Los Alamos

New 1D RT models that include the impact of 3D cloudiness

Given:

$$\begin{cases} I(z,\mu>0) = \varpi_0 \int_0^z \left| \frac{\mathrm{d}}{\mathrm{d}s} \left\langle T_0\left(s = \frac{z-z'}{\mu} \right) \right\rangle \right| \int_{4\pi} p(\mathbf{\Omega}' \to \mathbf{\Omega}) I(z',\mathbf{\Omega}')\mathrm{d}\mathbf{\Omega}' \frac{\mathrm{d}z'}{\mu} + Q(z,\mu) \\ I(z,\mu<0) = \varpi_0 \int_z^H \left| \frac{\mathrm{d}}{\mathrm{d}s} \left\langle T_0\left(s = \frac{z'-z}{|\mu|} \right) \right\rangle \right| \int_{4\pi} p(\mathbf{\Omega}' \to \mathbf{\Omega}) I(z',\mathbf{\Omega}')\mathrm{d}\mathbf{\Omega}' \frac{\mathrm{d}z'}{|\mu|} \end{cases}$$

where $Q(z,\mu) = (\mu F_0/\pi) \times \langle T_0(z/\mu)\rangle$ for $\mu > 0$, and 0 otherwise.

Find:

$$\begin{cases} T = \frac{2\pi}{F_0} \int_0^1 \mu I(z=H,\mu)\mathrm{d}\mu \\ R = \frac{2\pi}{F_0} \int_0^1 |\mu|\, I(z=0,-|\mu|)\mathrm{d}|\mu| \end{cases}$$

$$I_{\mathrm{zen}} = \pi I(z{=}H,\mu{=}{+1})/F_0$$
$$I_{\mathrm{nad}} = \pi I(z{=}0,\mu{=}{-1})/F_0$$